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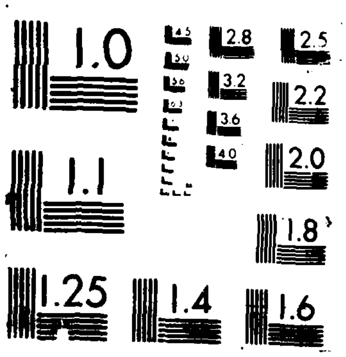
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A HANDBOOK OF SUPPLY INVENTORY MODELS

THESIS

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Major, USAF

AFIT/GLM/LSMA/87S-35

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A HANDBOOK OF SUPPLY INVENTORY MODELS

THESIS

Presented to the Faculty of the School of Systems and Logistics

of the Air Force Institute of Technology

Air University

In Partial Fulfillment of the

Requirements for the Degree of

Master of Science in Logistics Management

William C. Hood, B.A., M.S.

Major, USAF

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Preface

The purpose of this project was to develop a handbook on Air Force supply inventory models that would provide a ready reference for Air Force logisticians. The intended audience for this handbook is for the beginning student in supply operations at both base and depot levels. Therefore, I kept the theoretical discussion to a level so that the layman could easily understand the basics of the models.

I received assistance and advice from many people in conducting this research effort. First, I am deeply indebted to my thesis advisor, Captain Richard D. Mabe, for providing the expert guidance and constructive criticism that allowed this thesis to be sucessfully completed.

I am also especially grateful to the many other faculty members in the School of Systems and Logistics for their advice and unwavering support.

A special thank-you is due to the excellent library staff for helping me pull together all the research materials necessary for the thesis.

Finally, I want to express my deepest appreciation to my wife, Leonila and my daughter, Jennifer, who provided the moral support in getting me through the research effort and graduate program.

William C. Hood

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A HANDBOOK OF SUPPLY INVENTORY MODELS

I. Introduction

Chapter Overview

This chapter outlines a general background on the development and use of inventory models for the United States Air Force. It includes a statement of: the research problem, purpose, organization, scope and limitations.

Background

The Air Force has invested over nine billion dollars in expendable and recoverable items for the Air Force inventory. Expert management of these resources is a key element in our ability to build up and sustain combat capability. Historically, the Air Force has not managed aircraft assets effectively (4:1-13). Better use of techniques such as mathematical modeling not only offer opportunity for improved savings, but more important, the ability to increase the capability of the Air Force to mobilize and respond to world-wide threats.

The supply function involves primarily the management of two types of spare parts: recoverable and expendable spares. Though expendable spares generally are not very expensive, they comprise roughly 95 percent of the inventory at a typical base (5:5). Management of expendable spares using math models began in 1958. The Rand Corporation studied (for the Air Force) the possibility of using an Economic Order Quantity (EOQ) model which was being used extensively in private industry. The Rand study showed that a precise EOQ model would be difficult to obtain because of data collection costs. Therefore, they recommended that the Air Force adapt a more generalized approach using elements of the EOQ model (37:18). As a result, the Department of Defense directed in 1959 that all DOD activities use elements of the EOQ model in the management of inventory levels. The Air Force responded by integrating into their logistics system an hybrid EOQ model that is still in use today. The model sought to reduce total inventory handling costs through minimizing order and holding costs. Demand values input into the EOQ model are generated through a hybrid forecasting model developed during the 1960's (37:20).

Though expendable assets constitute the bulk of line inventory items, the bulk of asset costs lies in recoverable items. Unlike private industry, the Air Force is unique in owning a large inventory of recoverable items. Therefore, industry has no comparable model for the management of recoverable assets, such as the EOQ model they use for expendable assets.

The Repair Cycle Demand Level (RCDL) model is a basic pipeline model developed for use at the base supply level. This model calculates spares levels tailored to individual base repair capabilities as a result of the stockage policies used by base managers (12:7).

However, other models have been developed or are being developed to overcome the shortcomings of the RCDL model. The RAND Corporation proposed the Base Stockage Model in 1965 which was never implemented. In 1967, Sherbrooke developed the Multi-Echelon Technique for Recoverable Item Control (METRIC) model which successfully addressed management attention on the entire weapon system (36:122). Though this multi-item, multi-echelon, model was successfully tested in the field, it was later superseded by more sophisticated recoverable inventory models.

In 1973, Muckstadt introduced a modification to the METRIC model (MOD-METRIC) which allowed for a multi-indenture analysis of recoverable components in the basic METRIC model. MOD-METRIC is now used as the basic model in AFLC recoverable item management systems (32).

Although these recoverable inventory models were successful in measuring expected backorders and fill rates, it was difficult to translate this information into actual combat readiness of the fleet based on spare parts. As a result, the Logistics Management Institute created the LMI Availability Model in 1972. Their model measured aircraft availability as a function of demand and stock levels. This model turned the focus from <u>item</u> management to <u>systems</u> management in Air Force inventory analysis (16:6).

However, the LMI model only addressed the steady state system. In other words, the model could not measure surge or wartime demands on the inventory system. Concurrent with the LMI model, the Rand Corporation developed the Dyna-METRIC model in the 1980s which treated the complex and dynamic component repair process. AFLC Headquarters now uses Dyna-METRIC for assessing wartime capabilities. It will replace MOD-METRIC as the principle tool for recoverable item management in some management systems now being developed for AFLC.

Management training in mathematical inventory models is incomplete in the Air Force. Supply officers now receive training through an eleven week course offered by the Air Training Command at Lowry Air Force Base. The course is not a graduate level course in inventory theory, but rather an introductory course in supply management for officers in their first job at a base supply account. The students are taught the mechanics of the base inventory supply system, but not the theory for models used within supply (15:3.65).

Supply officers stationed at AFLC or AFSC may receive training in a particular inventory model, if their job is to determine requirements for spares to support a particular weapon system. Additionally, a course taught in the Professional Continuing Education Program at the Air Force Institute of Technology teaches the Dyna-METRIC model, primarily to Air Force and civilian workers at AFLC. Because inventory theory is such a technical and complicated subject, the study of supply inventory theory rightfully belongs at the graduate level. The Air Force Institute of Technology offers a supply officer the advanced theoretical training needed in the graduate inventory management option at the School of Systems and Logistics.

Problem Statement

There is now no specific text on Air Force inventory models that Air Force personnel can use to study inventory. Further, supply officers at all levels in the Air Force have no comprehensive reference source which explains the derivations, assumptions and uses of models they might use daily. Supply personnel could better manage the Air Force inventory system by understanding and working with complex inventory models. The Air Force needs a manual to explain the derivations, assumptions and uses of these models.

Statement of Purpose/Objectives

This thesis is a handbook on inventory models to be used by Air Force personnel for eduction and management. The models described are now in use in private industry, in base supply operations, and in the Air Force Logistics Command. Also included are forecasting methods that the Air Force uses to forecast demand rates and spares requirements. The handbook follows a standard format, with concise explanations and examples, and with standardized notation for all models. This thesis achieves three objectives:

(1) Collect information on the basic Wilson Economic Order Quantity (EOQ) Model and show how the Air Force derives its hybrid model for expendable spares.

(2) Research and present the inventory models used to model recoverable inventory pipelines.

(3) Research and present Air Force forecasting methods.

Organization of the Thesis

This project meets both the academic requirements of a Masters thesis and the practical requirements of a study manual. The customary thesis format includes a complete introduction, literature review, methodology, and documentation. This hankbook, however, will have an introduction chapter, followed by separate chapters for each class of inventory model. Appendix A covers Palm's theorem, the theoretical basis for recoverable asset models. Appendix B covers the basic performance measures used throughout the thesis. Appendix C is a collection of Dyna-METRIC formulas and an outline of the computer algorithm. Appendix D includes normal and poisson distribution tables to aid in understanding example problems.

Scope and Limitations

(1) The research and discussion of models for expendable items will be limited to the basic Wilson EOQ and the Air Force EOQ model derivation.

(2) The research and discussion on reparable inventory models will be limited to the base stockage model, METRIC,

MOD-METRIC, WARS, LMI Availability and Dyna-METRIC models, with application to the base supply level.

(3) The research and discussion on forecasting methods will be limited to basic time series methods, simple regression techniques, and some of the hybrid Air Force forecasting models.

(4) Air Force equipment management models will not be addressed in this manual.

II. EOQ MODEL

The Wilson EOQ inventory model is the earliest and most basic inventory model. It is widely used in private industry as well in the Air Force supply system. This chapter will first analyze the deterministic version of the EOQ model. Next, an algorithm for determining quantity discounts is discussed. The stochastic EOQ model is then analyzed with emphasis on determining backorder costs and service levels. Finally, the Air Force application of the stochastic EOQ model will be presented and analyzed.

List of Symbols and Abbreviations

BO	=	Backorder
C_B	=	Backorder cost per unit
C_H	=	Holding cost per unit
C_U	=	Cost per unit
D	=	Annual Demand
$\bar{D}$	=	Expected Annual demand
d	=	Lead time demand
$\bar{d}$	=	Expected lead time demand
EBPC	=	Expected backorders per cycle
FR	=	Fill Rate
N	=	number operating increments (days, weeks, etc.)
OST	=	Order and ship time
Q	=	Economic Order Quantity
R	=	Reorder point
S	=	Units backordered

SS = Safety Stock

TC = Total cost

V = Maximum inventory

DDR = Daily Demand Rate

VOD = Variance of Demand

VOO = Variance of Order and Ship Time

Deterministic EOQ Model

The classical EOQ inventory model is an idealized situation where total inventory costs for any particular item are minimized. This model, depicted in figure 1, computes the economic order quantity (Q), the inventory reorder point (R), and the order and ship time (OST) from order placement to stock receipt (20:454).

This classical model is based on the following assumptions (38:82):

(1) Demand rate is known and constant.

(2) Order and ship time is known and constant.

(3) Price per unit is constant.

(4) Order and holding cost per unit is fixed.

(5) Instantaneous receipt of order (i.e., no receipt processing required).

(6) No stockouts are permitted.

The objective of the EOQ model is to minimize total inventory costs per year. This total annual cost (TC) equals the purchase cost (C_u) for the annual inventory (D), plus order costs (C_o) for each order (D/Q), plus holding cost

($C_H C_U$) for the average inventory on hand (Q/2). Mathematically, this is:

$$TC = DC_U + C_O \frac{D}{Q} + C_H C_U \frac{Q}{2} \qquad (2.1)$$

The relationship of total cost to holding and order cost is shown graphically in figure 2. Holding cost per unit rises with greater inventory levels due to larger economic order quantities while order cost per unit will decrease.

The EOQ providing the least cost can be determined in two ways. First, the order cost can be set equal to the holding cost to determine Q, or:

$$C_O \frac{D}{Q} = C_H C_U \frac{Q}{2}$$

Then, Q can be derived algebraically:

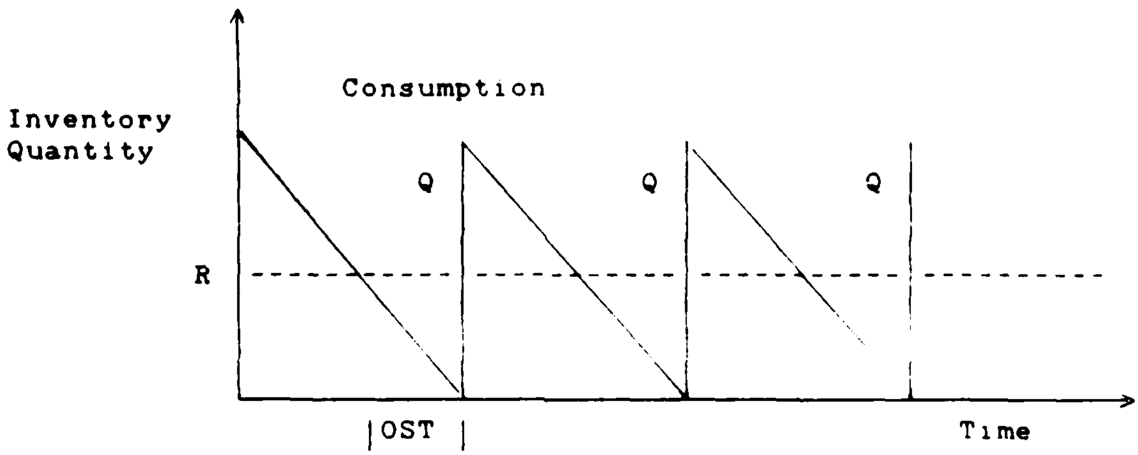
$$2C_O D = C_H C_U Q^2$$

$$Q^2 = \frac{2C_O D}{C_H C_U}$$

$$Q = \sqrt{\frac{2C_O D}{C_H C_U}} \qquad (2.2)$$

This finds Q at the point A on figure 2.

The second method for solving Q requires taking the first derivative of total cost with respect to Q and set it



R: Reorder Point
Q: Economic Order Quantity
OST: Order and Ship Time

Figure 1. Deterministic EOQ Model

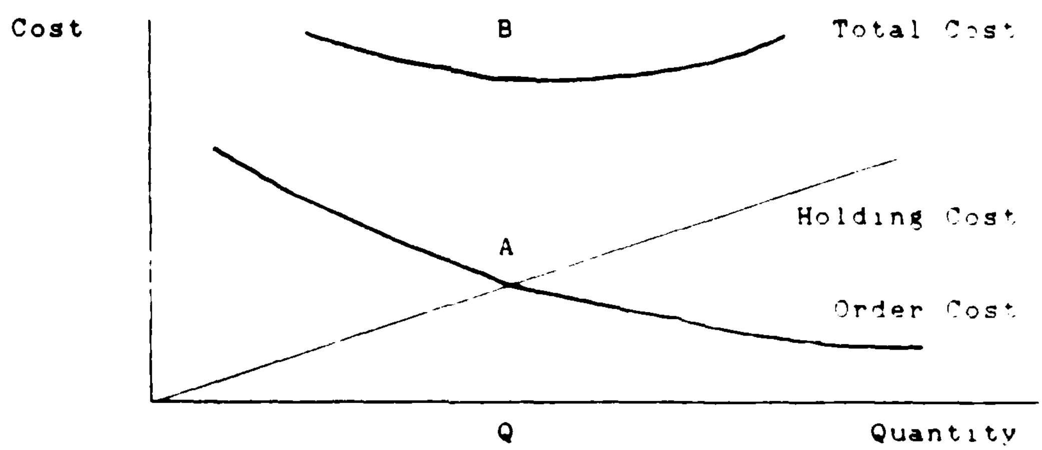


Figure 2. Derivation of Total Cost

equal to zero. This discovers a slope of zero on line TC (or point B).

$$TC = DC_U + C_O \frac{D}{Q} + C_H C_U \frac{Q}{2}$$

$$\frac{dTC}{dQ} = - C_O \frac{D}{Q^2} + \frac{C_H C_U}{2} = 0$$

$$Q^2 = \frac{2C_O D}{C_H C_U}$$

$$Q = \sqrt{\frac{2C_O D}{C_H C_U}}$$

This second method would be used when considering all costs involved, not just order and holding costs. Our example, however, only shows two costs.

Once the EOQ is determined, the reorder point (R), can then be determined by using the formula:

$$R = \frac{D(OST)}{N} \qquad (2.3)$$

where N is the number of operating days per year. The expected number of orders for the year is calculated as annual demands divided by the EOQ, or:

$$\frac{D}{Q} = \sqrt{\frac{DC_H C_U}{2C_U}} \qquad (2.4)$$

The average order interval is calculated as the EOQ divided by annual demand, or:

$$\frac{Q}{D} = \sqrt{\frac{2C_U}{DC_HC_U}} \tag{2.5}$$

<u>Backorder Costs.</u> If we allow stockouts to occur, then the backorder cost must also be included in the basic EOQ model. If backorder costs are high, then very few stockouts will occur while the reverse is true for low backorder costs. Graphically, this model is depicted in figure 3.

All the previous assumptions for the model in figure 1 hold true, except:

(1) Stockouts are allowed to occur.

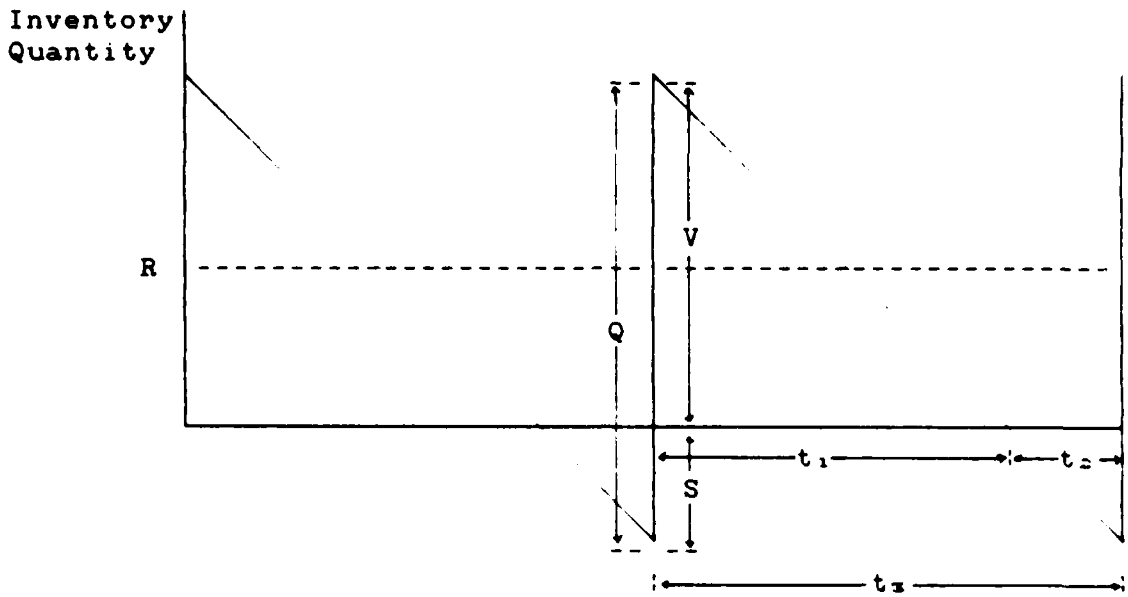
(2) All shortages are filled by the next lot quantity shipment (38:83).

In this model, the maximum inventory is equal to V while the size of stockout (S), is equal to Q-V.

Since the average inventory is now V/2, the holding cost is modified for a single time period (t_1) as:

$$C_HC_Ut_1\frac{V}{2}$$

Since the ratio of annual demand to one year (D/1) is equal to the ratio of maximum inventory to a specific time period (V/t_1), then:



Q = Economic Order Quantity
V = Maximum Inventory
R = Reorder Point
S = Stockout
t_1 = Time period from receipt of Q to stockout
t_2 = Time period from stockout to recipt of Q
t_3 = Time period from receipt of Q to next reorder of Q

Figure 3. EOQ Model with Stockout Condition

$$t_1 = \frac{V}{D}$$

and holding cost per year equals (38:84):

$$C_H C_U \frac{V}{2} \frac{V}{D} = C_C C_U \frac{V^2}{2D}$$

If the backorder cost per unit is C_B, the backorder cost for period t_2 is computed by multiplying the average inventory on backorder by the backorder cost per unit, or:

$$C_B \frac{(Q-V)}{2} t_2$$

Since the ratio of annual demand to one year (D/1) is equal to the ratio of stockouts to period t_2, or $(Q-V)/t_2$, then:

$$t_2 = \frac{(Q-V)}{D}$$

and backorder costs per year equals (38:84):

$$C_B \frac{(Q-V)^2}{2D}$$

Since purchase and order cost remain the same, the total annual cost is calculated as:

$$TC = DC_U + C_O \frac{D}{Q} + C_H C_U \frac{V^2}{2D} + C_B \frac{(Q-V)^2}{2D} \qquad (2.6)$$

By taking the partial derivatives of Q and V and setting them equal to zero, the optimal values are determined as:

$$Q = \sqrt{\frac{2C_O D}{C_H C_U}} \sqrt{\frac{C_H C_U + C_B}{C_B}} \qquad (2.7)$$

$$V = \sqrt{\frac{2C_O D}{C_H C_U}} \sqrt{\frac{C_B}{C_H C_U + C_B}} \qquad (2.8)$$

The reorder point calculation is modified to subtract the number of backorders (Q-V) so that:

$$R = \frac{D(OST)}{N} - (Q-V) \qquad (2.9)$$

Where N equals the number of operating days per year.

Quantity Discounts

The inclusion of quantity discounts complicates the model. The lower cost of a larger Economic Order Quantity might offset the added costs to handle more items. Thus, quantity discounts may be justified. The relationship between total cost, order costs, and holding costs is expressed in figure 4 (38:87). Quantity discounts do not affect order costs. Holding costs are reduced at each quantity discount because the unit cost (C_U) is reduced, thus reducing the value of holding, $(C_H C_U)(Q/2)$.

The minimum total cost occurs either at a point of discontinuity (points A or B), or at a point where the

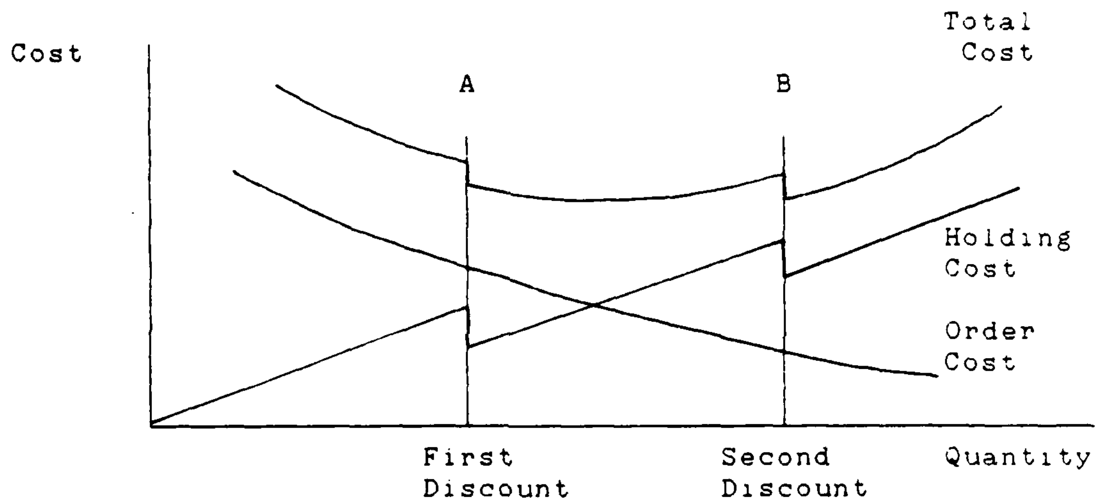


Figure 4. Effect of Quantity Discounts on Total Cost

derivative of TC with respect to Q is equal to zero, whichever is lowest. Figure 5 represents a decision algorithm to determine the total minimum cost where quantity discounts are involved. The valid Q is defined as the quantity equal to or greater than its price break quantity. In other words, the mathematically derived Q must fall within the range offered by the seller in order to receive the discount.

For example, if we were given the following information:

Lot Size	Unit Price
< 200	$12
200-399	$10
> 400	$ 9

where

Cost of Order (C_O) = 30

Cost of Holding (C_H) = .15

Annual Demand (D) = 3000

The first step would be to determine Q for the lowest price:

$$Q_9 = \sqrt{\frac{2(30)(3000)}{.15(9)}} = 365$$

The resulting Q (365 units) is not a valid Q because 365 does not fall in the range where the discount is offered (> 400) at the $9.00 purchase price. Therefore we calculate Q for the next lowest price of $10.00:

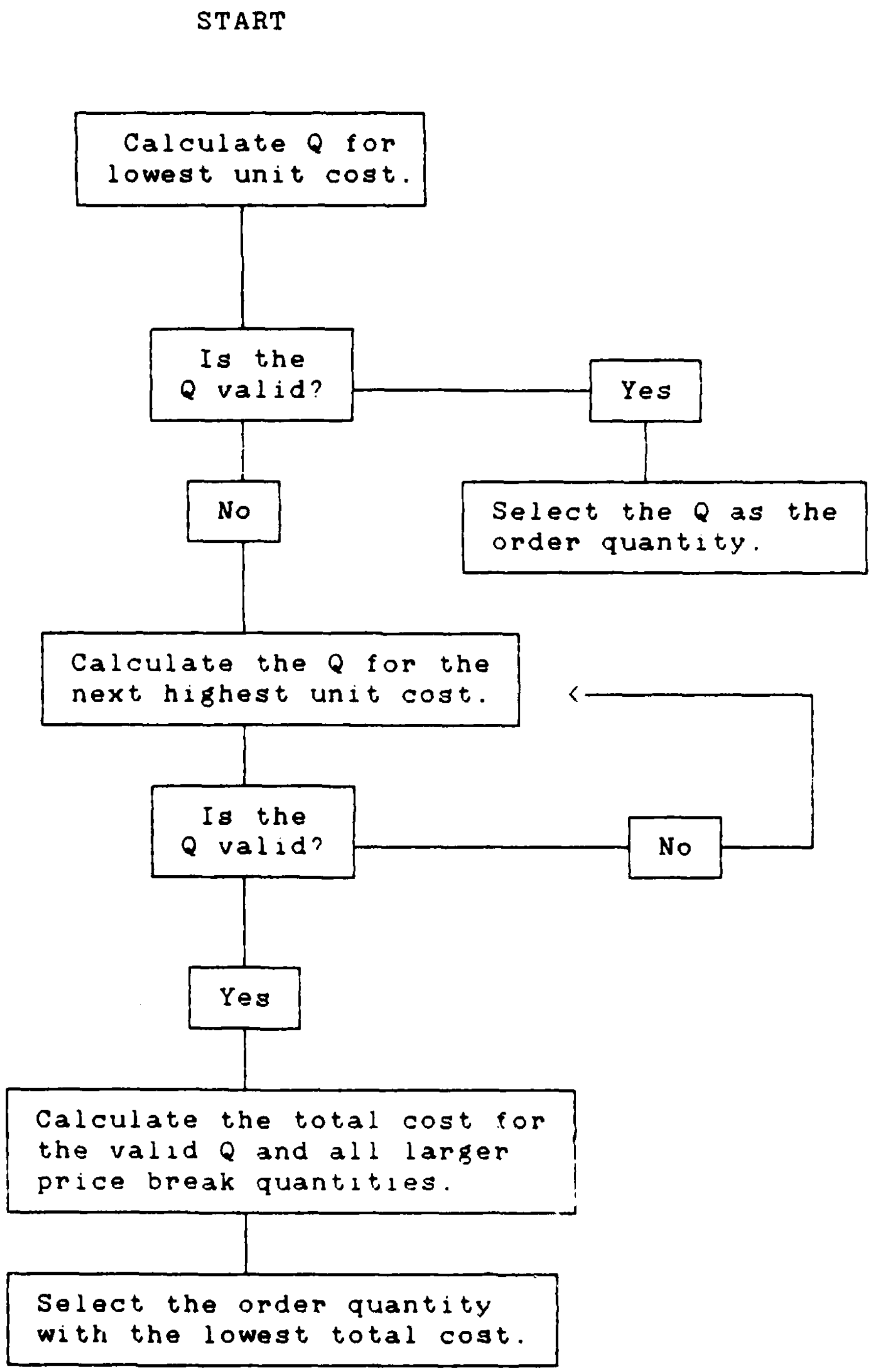


Figure 5. Quantity Discount Algorithm

$$Q_{10} = \sqrt{\frac{2(30)(3000)}{.15(10)}} = 346$$

In this case, the Q is valid because the Q falls within the lot size (200 - 399) corresponding to the $10.00 price. The final step is to compute the total cost for this price ($10.00), and the total cost for larger price break quantities (in this case, 400 units at $9).

$$TC_{346} = 10(3000) + (30)\frac{3000}{346} + .15(10)\frac{346}{2} = 30519.6$$

$$TC_{400} = 9(3000) + (30)\frac{3000}{400} + .15(9)\frac{400}{2} = 27495$$

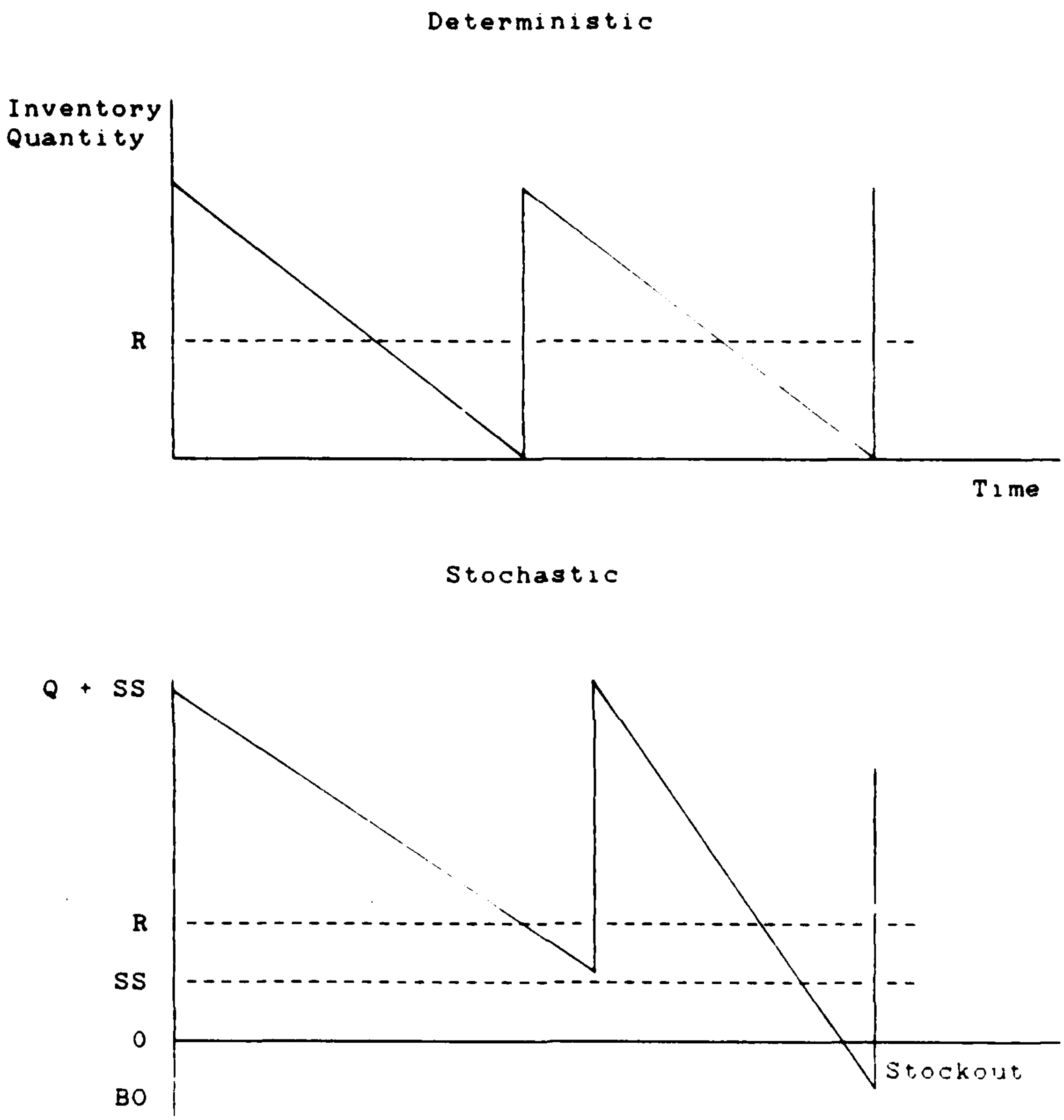
In this example, we would select the lowest total cost of $27495 and purchase 400 units at $9 each.

Stochastic EOQ Model

In reality, we find few cases where a deterministic EOQ model can be used because we cannot satisfy all of the assumptions of the deterministic model. Generally, a problem arises where demand and order and ship time (OST) rates are stochastic. Order and ship time can vary due to transportation and order problems, while demand can vary due to imperfect forecasting. Thus, an organization builds in a buffer of safety stock (SS) to protect against a stockout situation. If the cost of backorders is low, or the organization has a captive or loyal market, then the organization may elect to have low safety stock and allow stockouts to occur. If stockouts result in lost sales, or cannot be permitted for other reasons, then the organization must maintain high levels of safety stock to prevent stockout conditions. Safety stock will also be larger if holding costs are low, demand and order and ship time variations are large, and order and ship times are long. (38,136). Safety stock should be considered as a permanent investment (or sunk costs) by the organization.

Figure 6 demonstrates the difference between an ideal inventory model and a stochastic model. The primary difference is that total inventory held is Q + SS - BO. The inventory level will not decrease at a constant rate because quantities demanded vary over time. At the reorder point (R), Q level of inventory is ordered. If there is a higher demand during order and ship time, or if the time period is longer than usual, then safety stock (SS) is consumed to meet consumer demand. If safety stock is not adequate, as depicted in figure 6, then a stockout condition occurs. Once the replenishment stock arrives, backorders are filled prior to new customer demands.

The distribution most frequently used to describe demand and order and ship time variation is the normal distribution. Figure 7 demonstrates the fit of normal distribution to a stochastic EOQ model. The expected lead time demand (R - SS) is the mean of the normal lead time distribution. The shaded area of the normal distribution, 1 - F(x), is the cumulative



Q = Economic Order Quantity
R = Reorder Point
SS = Safety Stock
BO = Backorder

Figure 6. Comparison of Deterministic and Stochastic EOQ models

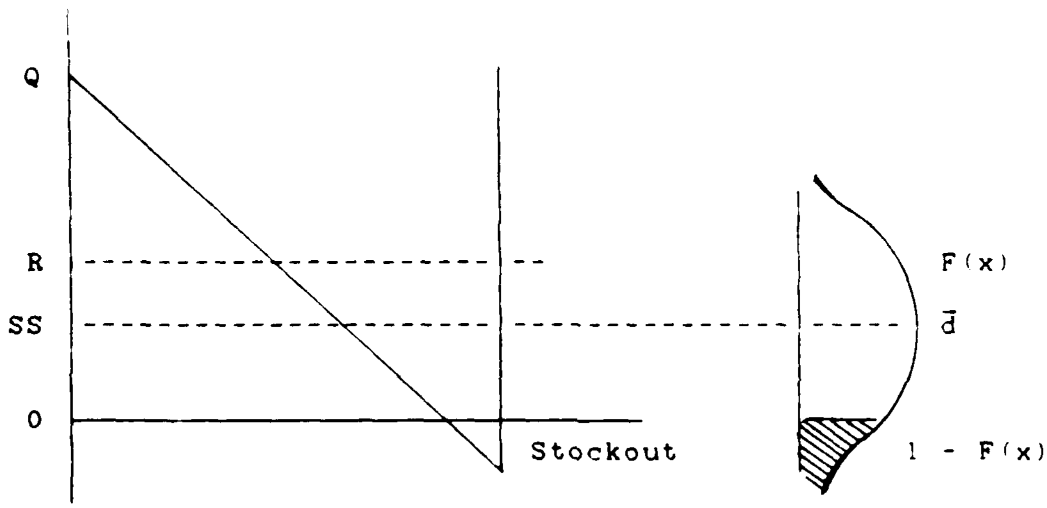


Figure 7. Fit of Normal Distribution
to EOQ Stochastic Model

probability of stockout if lead time demand is greater than the reorder point in terms of units.

The total annual cost of safety stock equals holding cost of the safety stock and stockout cost. If stockout costs are on a per unit basis, the formula used to determine the total annual cost of safety stock is:

$$TC_{SS} = C_H C_U (R - \bar{d}) + \frac{C_B D\ E(d > R)}{Q} \qquad (2.10)$$

where E(d > R) is the expected value of lead-time demand greater than the reorder point in number of units. By taking the derivative of the total cast with respect to the reorder point and setting it equal to zero, the optimum probability of a stockout with a known backorder cost per unit is:

$$1 - F(x) = \frac{QC_H C_U}{C_B D} \qquad (2.11)$$

By looking up F(x) in the normal distribution tables (Appendix D), the value Z can be extracted and safety stock can be determined if the variance of the distribution is known. The formula for determining safety stock is:

$$SS = \sigma Z \qquad (2.12)$$

and the reorder point can be calculated as:

$$R = \bar{d} + SS \qquad (2.13)$$

where $\bar{d}$ is the expected lead time demand or the mean of the lead time demand distribution. To determine the expected backorders per cycle (EBPC), the following formula is used:

$$EBPC = \sigma(E(Z)) \qquad (2.14)$$

where E(Z) is the partial expectation of Z. (Extract from partial expectation table in Appendix D). The fill rate (FR) can be determined by the formula:

$$FR = 1 - EBPC/Q \qquad (2.15)$$

If order and ship time is constant and lead time demand is normally distributed, the mean and variance of the lead time demand distribution are used in the determination of safety stock. If both the order and ship time and the lead time demand rates are normally distributed, then the combined mean is:

$$\mu = \mu_d * \mu_{OST} \qquad (2.16)$$

and the combined variance is:

$$\sigma^2 = (\mu_{OST})(\sigma^2{}_d) + (\mu^2{}_d)(\sigma^2{}_{OST}) \qquad (2.17)$$

Thus standard deviation is:

$$\sigma = \sqrt{(\mu_{OST})(\sigma^2{}_d) + (\mu^2{}_d)(\sigma^2{}_{OST})} \qquad (2.18)$$

However, in the Air Force, backorder costs are difficult to determine. The Air Force instead assigns an arbitrary service level in terms of the probability distribution of demand during lead time.

With this method, the desired service level, the cost per unit, holding cost, order cost, expected annual demand and the mean and variance of lead time demand should be known. An algorithm for determining safety stock, reorder point, expected backorder per cycle, and fill rate is as follows:

(1) Calculate Lot Quantity.

$$Q = \sqrt{\frac{2C_O D}{C_H C_U}}$$

(2) Calculate combined normal distribution of lead time demand and order and ship time.

$$\mu = (\mu_d)(\mu_{OST})$$

$$\sigma^2 = (\mu_{OST})(\sigma^2_d) + (\mu^2_d)(\sigma^2_{OST})$$

(3) Extract the Z from the normal distribution tables by entering with F(x). For example, if a service level of 85 percent is desired, then enter the tables with a F(x) of 85 percent.

(4) Calculate Safety Stock.

$$SS = \sigma Z$$

(5) Calculate Reorder Point.

$$R = \bar{d} + SS$$

(6) Calculate Expected Backorders Per Cycle.

$$EBPC = \sigma(E(Z))$$

(7) Calculate Fill Rate.

$$FR = 1 - \frac{EBPC}{Q}$$

<u>Example.</u> if a service level of 90 percent is desired and the following data is known:

$OST \approx n(10,3)$	$C_u = 8$
$d \approx n(14,2)$	$C_h = .15$
$D = 2000$	$C_o = 5.20$

1. Q is calculated as:

$$\sqrt{\frac{2(5.20)(2000)}{(.15)(8)}} = 131.6 \text{ or } 132 \text{ units}$$

2. The combined mean and variance of d and OST is n(140,62), or:

$$\mu = (10)(14) = 120$$

$$\sigma^2 = (10)(2) + (14)(3) = 62$$

3. The Z value is extracted by entering the normal distribution tables with .90 in the F(x) column (see appendix D):

$$Z = 1.28$$

4. SS = (140)(1.28) = 179.2 or 180 units

5. R = 140 + 180 = 320 units

6. EBPC = 7.874(.0475) = .374

7. FR = 1 -(.374/132) = .997 percent

Standard Base Supply System

The USAF Standard Base Supply System (SBSS) is an automated inventory accounting system used by all Air Force bases to control their supply functions. The system is characterized as a multi-item, single-echelon, continuous review inventory system with stochastic, multiple unit demands, backordering and an annual budget constraint (33:3). The current SBSS employs a variation of the classical EOQ formula as found in AFM 67-1, Volume II, Part Two (14:11-13). The objective of the formula is the same as the classical EOQ: that of minimizing the variable costs of holding and ordering. The reorder quantity is given by the EOQ formula while the reorder point is computed as the mean demand during lead time plus a safety level (8.19).

The Air Force uses a standard holding cost of 15 percent. For local purchases at base level (i.e. contracting), the cost of order is computed at $19.94 while non-local purchase (from depot) is computed at $5.20 per order. The EOQ formula for local purchase is:

$$EOQ = \frac{16.3\sqrt{DDR(365)(\text{Unit Price})}}{\text{Unit Price}} \quad \text{or} \quad \frac{16.3\sqrt{DC_U}}{C_U} \qquad (2.19)$$

The EOQ formula for non-local purchase is:

$$EOQ = \frac{8.3\sqrt{DDR(365)(\text{Unit Price})}}{\text{Unit Price}} \quad \text{or} \quad \frac{8.3\sqrt{DC_U}}{C_U} \qquad (2.20)$$

where DDR is the Daily Demand Rate. The computation of a DDR is discussed in chapter six.

If local purchase order cost is $19.94 and the holding cost is 15 percent, the classical EOQ formula yields:

$$Q = \sqrt{\frac{2(19.94)D}{.15C_U}} = \sqrt{\frac{(39.88)D}{.15C_U}} = 16.3\sqrt{\frac{D}{C_U}}$$

If we multiply the expression by C_u/C_u to avoid division in the radical, we get:

$$\frac{C_U}{C_U} * 16.3\sqrt{\frac{D}{C_U}} = \frac{16.3\sqrt{DC}_U}{C_U}$$

which is the Air Force EOQ formula. The same calculation holds true for the non-local order cost of $5.20:

$$\sqrt{\frac{2(5.20)D}{.15C_U}} = 8.3\sqrt{\frac{D}{C_U}} * \frac{C_U}{C_U} = \frac{8.3\sqrt{DC_U}}{C_U}$$

The reorder point (R) equals the Order and Ship Time Quantity (OSTQ) plus a safety level quantity (SLQ). OSTQ is defined as (14:11-13):

$$OSTQ = DDR(OST) \tag{2.21}$$

which is equivalent to the EOQ stochastic Eq (2.16). The SLQ is determined by the formula:

$$SLQ = C\sqrt{OST(VOD) + DDR^2(VOO)} \tag{2.22}$$

where

VOD = Variance Of Demand

VOO = Variance Of Order and Ship Time (σ^2_{OST})

C = service level factor (normally set at one)

In practice, "C" is the same as the value Z we extract from the normal distribution table when computing service level using the classical EOQ formula. Therefore, a C value of one equates to a service level of 84 percent while a C value of two equates to a 97 percent service level. This Air Force SLQ formula is equivalent to the classical EOQ Eq (2.12).

Example. If given the following information for a non-local purchase:

VOD = 3.5	C_u = 10
VOO = 20	C_h = .15
OST = 30	C = 1
DDR = .25	

EOQ, OSTQ and SLQ can be determined as:

$$EOQ = \frac{8.3\sqrt{(.25)(365)(10)}}{10} = 79.28$$

$$OSTQ = (.25)(30) = 7.5$$

$$SLQ = 1\sqrt{30(3.5) + (.25)^2(20)} = 10.307$$

EOQ Computation at AFLC

The Air Force Logistics Command (AFLC) is responsible for managing approximately 515,000 nonrecoverable line items which are officially catalogued with Expendability, Recoverability, Repairability, Category (ERRC) Code Designator of XB3 or XF3 (10:1). The management objective is to ensure maximum results in terms of supply availability and economy. AFLC manages these assets through five Air Logistics Centers (ALC) by using the D062 requirements computation system, which uses a modified EOQ system of minimizing variable costs of ordering, holding and backorders (3:12).

The EOQ model employed by AFLC can be characterized as stochastic, multiple item, single echelon, with allowable backorders and required safety stock. The D062 EOQ buy system is based on a periodic inventory review which is updated four times a month. Inventory items are stratified into Supply Management Grouping Codes (SMGC) which dictate how the items are managed and the degree of management intensity required (3:12). The reorder level of inventory items assigned to a SMGC can include the following parts: war reserve material (WRM), safety stock, lead time demand, depot supply level, and lag time demand (37:25). At the end of each month, each inventory item is considered for reassignment to a new SMGC by determining its dollar value of projected annual demand (PADR). The PADR is then calculated by determining net actual item price and net total demands (transfer, total sales, and nonrecurring (3:12).

The AFLC EOQ formula as found in AFLCR 57-6 is as follows (3:80-81):

$$Q = \sqrt{\frac{2AC}{H}} \tag{2.23}$$

where

Q = EOQ Dollar Value

A = Annual Demand

C = Cost to Order

H = Cost to Hold

By substituting the classical EOQ notation found earlier in this chapter, the AFLC EOQ formula is basically the same. Annual demand is calculated using actual unit price and the PMDR. The cost to hold and order varies among ALCs.

The safety level (SL) for any EOQ item is determined by the formula:

$$SL = K\theta \tag{2.24}$$

which is comparable to the classical EOQ Eq (2.16). However, the computation of SL is more involved. K is the safety factor in terms of number of standard deviations allowed while θ is the standard deviation of lead time demands. The computation of K and θ are based on a modification of a formula proposed by Presutti and Trepp in 1970 (34:243). The computation of θ is (3:80):

$$\theta = (PPR)^{.85}(.5945)MAD(.82375 + .42625LT) \quad (2.25)$$

where

PPR = Peacetime Program Ratio. A ratio used to calculate future inventory needs.

MAD = Mean Absolute Deviation. The difference between a quarter's forecasted demand and the actual average.

LT = Lead Time. A function of PMDR, administrative, and production lead times.

.5945 = Constant which converts the mean absolute deviation from a quarterly to a monthly value.

.82375 and

.42625 = Constants which expresses the MAD over lead time and recognizes that a particular month's demands are influenced by a previous month's demands.

The formula for K is (3:80):

$$k = -.707\ln \left[\frac{\sqrt{2}\ (HC)\ (Q)\ (UC)}{\frac{\lambda}{\sqrt{R}}\ (\theta) \left(1 - \left(\exp \frac{\sqrt{2Q}}{\theta}\right)\right)} \right] \quad (2.26)$$

where

HC = Holding cost

Q = EOQ

UC = Actual Unit Cost

R = Average Requisition Size

θ = Standard Deviation of Lead Time Demands

exp = Exponential function

ln = Natural Logarithm

λ = Implied Shortage Factor

The implied shortage factor λ is a mathematical expression used to adjust the safety level in order to meet budget constraints for a specific time period. In other words, it can establish a safety level to meet a desired budget or readiness goal. The values of λ, K, and SL are all positively correlated. An increase in λ will cause an increase in safety level.

Summary

This chapter first analyzed the basic Wilson EOQ model with both deterministic and stochastic characteristics. Applications of the model can effectively reduce the cost of carrying inventory. The Air Force has extensively used variations of this model to manage the large amount of required inventory. Specifically, the Air Force uses EOQ in the management of nonrecoverable items at both the base and depot levels.

III. Repair Cycle Demand Level Inventory Model

The repair cycle demand level (RCDL) model is the basic pipeline model used to manage reparable assets in the Standard Base Supply System (SBSS). These assets can usually be characterized as high cost, low demand type items comprising 95 percent of all money spent on supplies at a typical base (5:5). This chapter will describe the characteristics of the model, build the model from a basic deterministic version to the full stochastic model, and finally give examples to how the model works.

System Description

The RCDL model uses the (S-1,S) continuous review inventory policy. This policy means that whenever a demand for an arbitrary number of units is accepted (S-1), a reorder is placed immediately for that number of units. This restores the total of stock on hand plus on order minus backorders to the spare stock level, S (18:1).

When a reparable item fails and cannot be repaired on the aircraft, then flightline maintenance removes the item and takes it to shop maintenance for repair. At the same time, a replacement item is ordered from base supply, delivered, and installed on the aircraft. This begins the repair cycle time (RCT) process. Depending on the ERRC code, technical order specifications, and maintenance capability, the item is either repaired at the base level or declared Not Repairable This Station (NRTS).

If repaired at base level, the item is turned back into base supply to replenish shelf stock. If NRTS'd, the item is routed to base supply which in turn sends the item to a depot or contractor for repair. When turned-in to base supply, (whether repaired or NRTS), the RCT ends. If the item is routed to depot for repair, a requisition is made against depot stocks for a like item to bring base level shelf stock back to equilibrium. A graphical depiction of the system is shown in figure 7.

Deterministic One Echelon Model

If the model is limited to base level only, and deterministic only, then it must meet the following assumptions (11):

(1) All reparable items are repaired at base level. This implies a percentage of base repair (PBR) equal to one.

(2) No variability in DDR or RCT.

(3) All items are repairable.

The total stock required (S) at base level for any particular reparable item can be expressed as the RCT multiplied by the DDR, resulting in the repair cycle quantity (RCQ) (11):

$$S = DDR * RCT = RCQ \tag{3.1}$$

The RCQ will maintain the system in equilibrium with no shortages occurring.

For example, if the DDR for an item is three units per day, and the RCT for that item is three days, the total

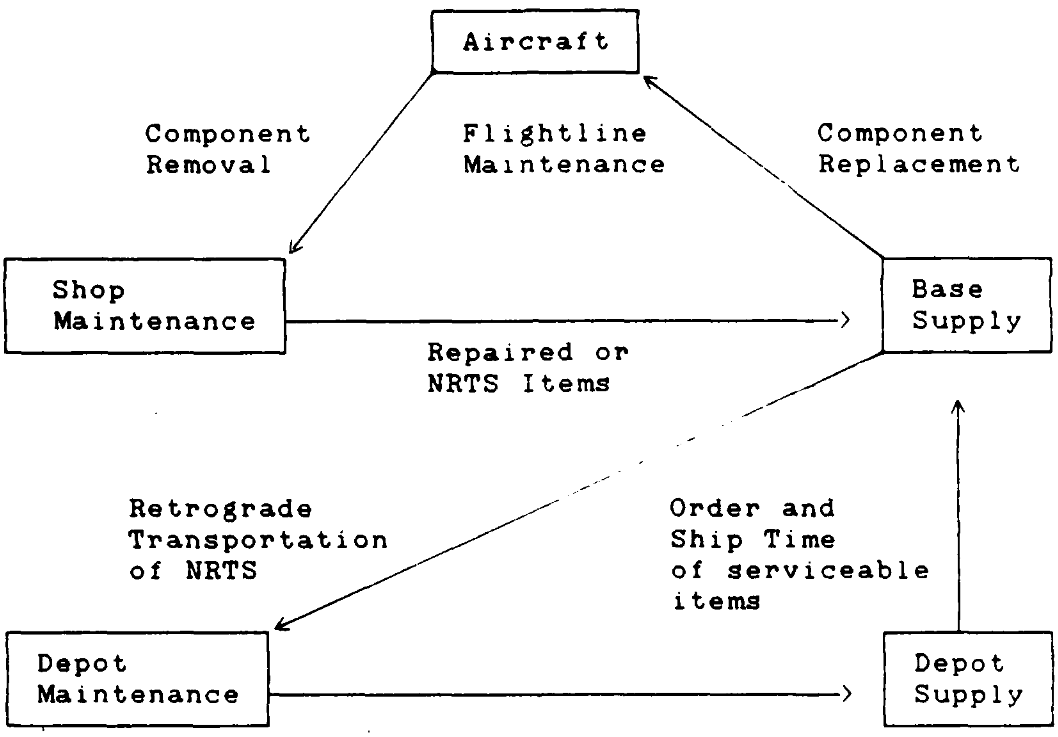


Figure 8. Repair Cycle

system stock required to keep all aircraft in operation would be nine units.

Deterministic Two Echelon Model

If the second echelon (depot level) is included in the model, two other factors are included in the basic formula. When a reparable item is NRTS'd back to the depot, a requisition is made on depot stock for a like replacement. The time of order to the time of arrival at base supply is called the order and ship time (OST). Since NRTS items are those items not repaired at base level, the NRTS figure is expressed as the percentage of items that are not repaired at base level, or (1 - PBR). If (NRTS * OST) is multiplied by the DDR, the result is the order and ship time quantity (OSTQ). When combined with the RCQ, the result is the total stock required to keep the system in equilibrium. The equation becomes (11):

$$S = RCQ + OSTQ$$

or

$$S = DDR[(RCT * PBR) + (OST * NRTS)] \qquad (3.2)$$

However, one more factor must be included in the equation. The decision to NRTS an item is not instantaneous. The period of time from arrival of failed unit to the maintenance shop to decision time is known as NRTS/condemn time (NCT). Therefore, the system will require more units to maintain equilibrium to account for the NCT. If (NRTS * NCT) is multiplied by the DDR, the result is the NRTS condemn time quantity (NCTQ). When combined with the RCQ and OSTQ, the result is the total stock required to keep the system in equilibrium. The equation becomes (11):

$$S = RCQ + OSTQ + NCTQ$$

or

$$S = DDR[(RCT * PBR) + (OST * NRTS) + (NCT * NRTS)] \qquad (3.3)$$

For example, if the DDR for an item is three units per day, RCT is three days, PBR is 25 percent, OST is ten days, NCT is six days, the total stock required is:

$$S = 3[(3 * .25) + (10 * .75) + (6 * .75)] = 38.25 \text{ units}$$

Stochastic Two Echelon Model

To account for variability in the model, the pipeline model adds a safety level quantity (SLQ) to achieve a desired service rate. The model assumes a normal distribution with variance equal to three times the mean quantity (3S), (or a variance to mean ratio of three to one). To achieve an 84 percent service rate, a C factor or one standard deviation (square root of 3S) is added to the mean quantity S (12:8). Increasing the C factor increases the service level. The formula is therefore (14:11-13):

$$SLQ = C * \sqrt{3 * (RCQ + OSTQ + NCQ)} \qquad (3.4)$$

If for example, we used the same figures as given in the example for the deterministic two echelon model, the SLQ computed would be 6.18 with a C factor of one. This would raise the stock level from 38.25 units to 44.43 units in order to achieve an 84 percent service level.

In addition, AFM 67-1 adds a constant adjustment factor K to the model. This adjustment is for rounding purposes only. The constant is .5 if the unit price is greater than \$750 or .9 if the unit price is less (14:11-13). Therefore, the complete RCDL model is:

$$S = RCQ + OSTQ + NCTQ + SLQ + K \qquad (3.5)$$

AFM 67-1 also adds a variation to the model when the unit price is less than \$750 and the PBR is less than 50 percent (14:11-13). In this case, an EOQ is determined for the item (as explained in Chapter Two) and added to the model resulting in:

$$S = EOQ + RCQ + OSTQ + NCTQ + SLQ + K \qquad (3.6)$$

Summary

The ubiquitous RCDL model is characterized as single item, single indenture, one location, two echelon, multiple period, and stochastic. The algorithm for determining stock levels is relatively straight-forward. However, the model does have weaknesses in that: (1) it treats each item independently of all other items, (2) it does not take cost into account, and (3) it does not give any indication of weapon system performance (25:21).

IV. Backordered Centered Models for Recoverable Assets

Overview

The repair cycle demand level (RCDL) model was developed in the early 1960's and is still in use today at the base or operating level. There have also been efforts to design recoverable asset models for multi-base and depot applications. This chapter will show the development of backorder centered models, including the Base Stockage Model (BSM) developed by the Rand Corporation in 1965, the Multi-echelon Technique for Recoverable Item Control (METRIC) model proposed by Sherbrooke in 1967, and finally a modified METRIC model (MOD-METRIC) developed by Muckstadt in 1973.

All of these models have similiar characteristics. First, these models incorporate Palm's theorem which states: if demands arrive (at a service queue) according to a poisson process, then the number of items in resupply is also poisson for any arbitrary distribution of demands (13:5). Appendix A covers Palm's theorem in more detail.

Second, this class of models uses expected backorders as a performance measure, or:

$$E(B|S_i) \quad \sum_{X=S+1}^{\infty} (X - S)\,p(X|\lambda T) \qquad (4.1)$$

where S is beginning stock , X is the quantity of beginning stock demanded, λ is the mean demand rate, and $p(X|\lambda T)$ is the probability of observing X demands during the time period being measured (30:4). A full explanation of performance measures is given in Appendix B.

Third, these three models represent a steady-state situation, which means the demand rate and its associated variation remain constant over time. As a result, these models are more appropriate for peace-time rather than war-time use. Chapter Five will address non-steady state models.

Base Stockage Model (BSM)

Feeney and Sherbrooke criticized the RCDL model saying it ignores unit cost. In other words, two items with the same demand characteristics, but with differing unit prices, will receive the same stock level under the RCDL computation. Feeney and Sherbrooke argue that a more optimal policy is to create a model that stocks more units of a low cost item at base level while relying on premium transportation to expedite from depot to base level for high cost items (19:3). The BSM uses a systems approach. All items in base supply are examined, and trade-offs are made between all items in order to maximize a system objective, subject to a cost constraint (19:10-11).

While building the BSM, Feeney and Sherbrooke discovered a large variability in demand distributions at base level which implies Palm's theorem will not work. They believed that this large variability was due to a compound poisson process, or a process where the variance can be equal to or greater than the mean and still use Palm's theorem. If the variance equals the mean, then the compound poisson process would be reduced to a simple poisson process (18:6). A compound poisson process in Air Force supply operations means a part may receive multiple, simultaneous demands at any given time. Reasons these multiple demands occur include (18:6):

(1) failure of an item on one aircraft will lead to further inspections of other aircraft for like failures.

(2) some items will have a high failure rate initially after installation.

(3) items damaged during installation lead to increased demand later during use.

Feeney and Sherbrooke demonstrated that Palm's Theorem for a compound poisson process can be shown as:

$$p(X) = \sum_{Y=0}^{\infty} \frac{(\lambda T)^{Y} \quad e^{-\lambda T}}{Y!} \quad f^{Y*}(X) \qquad (4.2)$$

where $f^{Y*}(X)$ = the Y-fold convolution of f, and f is the probability that Y customers place a total of X demands (18:7). λ is the arrival (or failure) rate, and T is the number of units in the service queue. Mathematically, λT is described as follows (assuming Palm's Theorem applies):

λ = DDR (the daily demand rate for an item.)

T = [(PBR * RCT) + (NRTS * OST)] or the number of items in maintenance after failure.

thus: Pipeline = λT

where

PBR = Percent Base Repair

RCT = Repair Cycle Time

NRTS = Not Repairable This Station

OST = Order and Ship Time

(Note: In this model, one failure = one demand for a replacement.)

Model Presentation. The objective of the BSM is to minimize the number of expected backorders subject to a budget constraint. Mathematically, this equates to:

$$\text{Minimize} \sum_{i=1}^{n} E(B_i) \tag{4.3}$$

subject to

$$\sum_{i=1}^{n} C_i S_i \tag{4.4}$$

where

$E(B_i)$ = Expected backorders for i items

C_i = Cost of i items

S_i = Stock of i items

The method of achieving the stated goal is through marginal analysis. The model assumes an initial zero stock level for every item, then computes which items would give the most fill protection (i.e., fewest back orders) per dollar of stock (17:14-15). Units are added to stock levels as they provide the greatest reduction in total backorders per dollar spent. Mathematically, this is accomplished through analysis of a benefit (reduced backorders) to cost ratio computed as (11):

$$\frac{\text{Benefit}}{\text{Cost}} = \frac{E[B_i|S_i] - E[B_i|(S_i + 1)]}{C_i} \qquad (4.5)$$

<u>Example.</u> To demonstrate how the model works with three items and a total budget of $26, the following information is given:

Item #:	1	2	3
T:	2	3	4
Cost:	$4	$3	$2

Table 1 shows expected backorders for three items for each stock level from zero to six. When the stock level is equal to zero, the expected backorders for each item equals the λT for that item. If S equals one, the expected backorders equal .8647 for item one. All values were extracted from cumulative poisson tables (Appendix D). These expected backorders given a stock level of one (.8647) are then subtracted from the expected backorders given a stock level of zero, (2.0), resulting in a marginal decrease to 1.135 units. The expected backorders for item one with a stock level of two is .5940, resulting in a marginal decrease to .541 units when subtracted from the previous figure of

Table 1. Marginal Return Table

	(Expected Backorders $\mid S_n$)		
Stock level (S_n)	Item One	Item Two	Item Three
0	2.0	3.0	4.0
1	1.135	2.049	3.018
2	.541	1.249	2.109
3	.218	.672	1.348
4	.075	.319	.781
5	.022	.135	.410
6	.006	.051	.195

Computed using $E[B_i \mid S_i] - E[B_i \mid (S_i + 1)]$

Table 2. Benefit/Cost Computation

	Benefit/Cost		
Stock Level	Item One	Item Two	Item Three
0	.5	1.0	2.0
1	.284	.683	1.509
2	.135	.416	1.055
3	.054	.224	.674
4	.019	.106	.391
5	.006	.045	.205
6	.002	.017	.098

Table 3. Allocation of Items

Allocation	Item One	Item Two	Item Three	$\Sigma S_i C_i$
1	0	0	1	2
2	0	0	2	4
3	0	0	3	6
4	0	1	3	9
5	0	2	3	12
6	0	2	4	14
7	1	2	4	18
8	1	3	4	21
9	1	3	5	23
10	1	4	5	26

1.135 units. The rest of the table is computed in the same manner (11).

Next, to complete the benefit/cost equation, all figures in table one are divided by the cost of the item, resulting in table two.

The last step is to allocate each item with the highest benefit/cost ratio to stock until the constraining value of $26 is spent. In other words, the item that minimizes the most backorders at the lowest cost is picked first where S = 0, or item 3. Now the ratios are recomputed at S = 0 for items 1 and 2 and S = 1 for item 3. Again, item 3 is picked. This continues, letting S increase for each item and selecting the lowest benefit/cost ratio for the stock on hand. Table three on page 48 demonstrates the order and number of items picked.

As can be seen, the model favors low cost items. Only one of item one will be stocked while five of item three will be added to the inventory. The BSM model in theory works better than the current pipeline model now in use at base level supply because the BSM optimizes expected backorders. Theoretically, the BSM model should attain the same performance as the pipeline model with less than one-half the investment cost for spares.(19:23).

In summary, the BSM can be characterized as multi-item, single indenture, single location and single echelon.

METRIC Model

In 1967, Sherbrooke improved on the Base Stockage Model (BSM) by enlarging its structure to cover multiple base locations, and include the depot level. This corrected the primary limitation of the BSM, in that the BSM only optimized the system at a single base and ignored the dependent influence of the depot and other bases on a single base supply system.

This new model, called the Multi-Echelon Technique for Recoverable Item Control (METRIC), has three purposes. First, it can be used to determine base and depot stock levels so that the sum of the expected backorders is minimimized at all bases having a particular weapon system. Second, the model can be used to determine stock levels for each particular item that minimizes the expected total base backorders. Last, the model can be used for analysis of system performance (30:1-2).

The advantages of the METRIC system include (36:124):

(1) METRIC uses the same mathematical formulation as the BSM, therefore, experience gained from using the BSM can be directly applied to the METRIC model.

(2) METRIC uses past data, but combines them with estimates of future requirements to anticipate build-ups or phase-outs.

(3) METRIC allows for a smooth transition from initial support planning to follow-on provisioning.

(4) METRIC allows for easy evaluation of the impact of different maintenance policies or pipeline times on the supply system.

(5) METRIC allows management to provide different levels of support to different weapon systems.

Assumptions. The METRIC model operates under the following assumptions (36:129-130):

(1) The distribution of demand over a period of time is stationary. This characterizes METRIC as a steady-state model.

(2) Lateral resupply between bases is ignored.

(3) No condemnations are allowed.

(4) Base and depot repair begins immediately when a broken recoverable item arrives at the shop or depot. Items are not batched for repair at either location.

(5) Items are normally considered to be equally essential.

(6) Demand data from different bases can be pooled to arrive at one estimator for an overall demand rate.

As with the BSM, the METRIC model also uses a compound poisson process to explain the demand on the system. However, METRIC uses a logarithmic poisson process which is a member of the compound poisson distribution family, whereas the BSM incorporated a geometric poisson. The logarithmic poisson is a process where a batch of demands arrives according to a poisson distribution, but the number of demands per batch follows a logarithmic distribution

(36:128). Sherbrooke argures for the logarithmic poisson because the state probabilities (Probability of "n" demands in a "t" time-interval), are negative binomial, which is easy to compute (36:128).

<u>Model Presentation.</u> To determine stock levels at different bases (given a depot stock level), the sum of the expected backorders for recoverable items is minimized. The first step in this process is to compute the effect of depot backorders on the system. Depot backorders are only considered as a factor in how they affect base backorders (36:126).

Recall in the BSM model, λT was computed where:

$$\lambda T = [(PBR * RCT) + (NRTS * OST)]$$

In the METRIC model, though, Depot Delay Time (DDT) must be included in the formula to account for delay due to depot stock shortages. If the depot had an infinite supply of stock, then DDT would be zero, and the OST would account for all administrative and pipeline times (30:4). If the depot carried no stock, then DDT would equal the average depot repair time for that item (D). Therefore, the equation for λT needs expansion to:

$$\lambda T = [(PBR * RCT) + (NRTS(OST + DDT))] \qquad (4.7)$$

DDT is determined using the same compound poisson process found at the base level. The expected number of units delayed at the depot at some arbritary point of time is (30:5):

$$B(S_o \mid \lambda D) = \sum_{X=S_o+1}^{\infty} (X - S_o)p(X \mid \lambda D) \tag{4.7}$$

where

S_o = depot stock

X = demands

D = average depot repair time

λ = $\Sigma\lambda_j(NRTS)$, λ_j = monthly demand rate at base j and

$NRTS_j$ = percentage of units NRTS at base j.

If $B(S_o \mid \lambda D)$ is divided by λ, the result is the average delay per demand measured in months. If we define:

$$d(S) = \frac{B\ (X \mid \lambda D)}{D}$$

Then $d(S) * D$ is the average delay per demand, or DDT. For example, if $D = 4$, $\lambda = .5$, and $S_o = 5$, then:

$$d(S) = \frac{(X - S)p(X \mid \lambda D)}{D} \tag{4.8}$$

Using the poisson tables (Appendix D), we find that $d(S)$ = .2177, therefore DDT = (.2177) * 4 = .8708.

Further recall the objective function of METRIC is to minimize the expected backorders at all bases given a set depot stock level. Mathematically, this is described as:

$$\text{Minimize} \quad \sum_{i=1}^{n} \sum_{m=1}^{m} E(B_{ij} \mid S_{ij}) \tag{4.9}$$

Subject to

$$\sum_{i=1}^{n} C_i (S_{io} + \sum_{j=1}^{m} S_{ij}) \leq \$ \text{ constraint} \tag{4.10}$$

where

i = item

j = bases

C_i = cost of item

S_{ij} = Stock level of i items at base j

S_{io} = Depot stock levels of item i

As with the BSM, marginal analysis using a benefit/cost ratio is used to determine which items are stocked at each base.

<u>Example.</u> Given a constraining budget value of $33, and the following information, the allocation of items to the bases can be determined.

	Depot			Base One			Base Two	
Item:	1	2		1	2		1	2
Stock:	1	2	T:	1	2	T:	2	3
Cost:	$3	$2	Cost:	$3	$2	Cost:	$3	$2

First, compute a marginal return table for each base (Table 4). Next, divide all figures in table 4 by the cost of each item to arrive at the benefit/cost ratio (table 5).

Table 4. Marginal Return Table

Base One			Base Two		
Stock Level	Item One	Item Two	Stock Level	Item One	Item Two
0	1.0	2.0	0	2.0	3.0
1	.368	1.135	1	1.135	2.049
2	.104	.541	2	.541	1.249
3	.023	.218	3	.218	.672
4	.004	.075	4	.075	.319
5	.001	.022	5	.022	.135

Table 5. Benefit/Cost Computation

Base One			Base Two		
Stock Level	Item One	Item Two	Stock Level	Item One	Item Two
0	.333	1.0	0	.667	1.5
1	.123	.568	1	.378	1.025
2	.035	.271	2	.180	.625
3	.008	.109	3	.073	.336
4	.001	.036	4	.025	.160
5	.001	.011	5	.007	.068

Table 6. Allocation of Items

Base One			Base Two		
Allocation	Item One	Item Two	Item One	Item Two	$\Sigma C_i S_i$
1	0	0	0	1	2
2	0	0	0	2	4
3	0	1	0	2	6
4	0	1	1	2	9
5	0	1	1	3	11
6	0	2	1	3	13
7	0	2	2	3	16
8	0	2	2	4	18
9	1	2	2	4	21

Finally, allocate each item to each base with the highest cost/benefit ratio until $33 is reached. If the cost of stock at the depot is $12, then the allocation at the bases is limited to $21 (Table 6).

Summary. As can be seen, the METRIC model will still allocate low cost items to the bases first just as in the BSM. However in the METRIC model, this allocation is distributed among all bases. In summary, METRIC can be characterized as multi-item, single indenture, multi-location and multi-echelon. The METRIC model was initially incorporated into the D041 system at AFLC to compute item requirements, however, the model was replaced in 1983 (25:26).

MOD-METRIC Model

One of the shortcomings of METRIC caused the model to buy inexpensive recoverable subcomponents, rather than buy the more expensive component items. Muckstadt in 1973 introduced a modification of the METRIC model to correct this shortcoming by explicitly considering the hierarchical parts structure. He established an indenture relationship between components and their subcomponents. The components are called Line Replacement Units (LRU) while the subcomponents are called Shop Replacement Units (SRU) (25:28).

The relationship between a LRU and its SRUs is described in the following manner. A defective LRU on an aircraft is assumed to ground that aircraft and is the result of a SRU failure within the LRU. Maintenance technicians will remove the LRU from the airframe to the shop for repair. Then a replacement LRU is ordered from supply and installed on the airplane. In the shop, the defective SRU is removed from the LRU and replaced with a unit ordered from supply. A backorder for an LRU will directly effect the operational mission by grounding aircraft, while a backordered SRU will only delay the repair of the LRU (32:475).

<u>Assumptions.</u> All of the METRIC assumptions apply to the MOD-METRIC model except for one. In METRIC, all items are considered to be equally essential. In MOD-METRIC, this assumption is inappropriate, because of the different impact on performance of an LRU and a SRU. In addition the following assumptions hold (11):

(1) Each LRU failure is due to only one SRU failure.

(2) Each SRU belongs to only one LRU.

(3) LRUs are normally repaired at base level while SRUs are repaired at the depot.

<u>Model Presentation.</u> As with the METRIC model, the expected number of units delayed at the depot is the same as eq 4.7 (32:476). If $B(S_o|\lambda D)$ is divided by λ, the yield is the DDT.

While METRIC computes the average number of units in resupply as,

$$\lambda T_i = (PBR_i \quad RCT_i) + NRTS_i(OST_i + DDT_i)$$

MOD-METRIC computes the average number of LRUs in resupply as:

$$\lambda T_i = PBR_i(RCT_i + SDT_i) + NRTS_i(OST_i + DDT_i) \quad (4.11)$$

where SDT_i is the average delay in base repair due to the unavailability of a SRU.

The expected delay in engine repair at base i due to SRU unavailability is (32:476):

$$SDT_i = \frac{1}{PBR_i} \sum_{j=1}^{n} \lambda_{ij}\Delta_{ij} \quad (4.12)$$

where

n = Number of SRU

λ_i = Removal rate for LRUs at base i

λ_{ij} = Average number of daily removals of SRU j at base i

Δ_{ij} = Expected delay in LRU base repair time due to a backorder on SRU j at base i

The expected delay in LRU base repair time ($_{ij}$) is computed as (32:476):

$$\Delta_{ij} = \frac{\sum_{X_{ij}=S_{ij}+1}^{\infty} (X_{ij}-S_{ij})\,p(X_{ij} \mid \lambda_{ij}T_{ij})}{\lambda_{ij}} \quad (4.13)$$

where

T_{ij} = Average resupply time for SRU j at base i.

The objective of the MOD-METRIC model is to minimize expected base backorders for all end item subject to a dollar constraint on the LRU and SRU. Mathematically, this is:

$$\text{Minimize} \quad \sum_{i=1}^{m} \quad \sum_{X_i = S_i + 1}^{\infty} (X_i - S_i) \; p(X_i \mid \lambda T_i) \qquad (4.14)$$

subject to

$$\sum_{i=1}^{m} C_e S_i \sum_{j=1}^{n} C_j S_{ij} + \sum_{j=1}^{n} C_j S_{oj} + C_e S_o \leq \$ \text{ constraint} \qquad (4.15)$$

where

S_i = Stock level of spare engines at base i

C_e = Unit cost of an LRU

C_j = Unit cost of SRU j

Summary. MOD-METRIC can be characterized as a multi-item, multi-indenture, multi-location and multi-echelon model. The model was designed specifically for the management of F-15 aircraft engines and their subcomponents. These engines have for the most part a modular design, where the vast majority of recoverable items are located in the modules (32:473). MOD-METRIC is therefore well suited for the management of these items.

Conclusion

There are two primary shortcomings to backorder centered models. First, the Base Stockage Model, METRIC, and MOD-METRIC use expected backorders as a performance measure. While expected backorders may be the best measure of the direct category, (see Appendix B), operational performance measures are more readily understood by Air Force managers, such as Not-Mission-Capable (NMC) aircraft or Fully-Mission Capable (FMC) sorties.

Second, this class of models only computes steady-state systems. While some aspects of Air Force supply might fit this criteria, a dynamic model would be more appropiate to fulfill the requirements of a war-time environment. The next chapter will address models that correct these two deficiencies.

V. Availability Centered Models for Recoverable Assets

Overview

The previous chapter covered models that used expected backorders as a performance measure. This chapter will address models that use operational availability criteria as a performance measure. These performance measures directly measure the impact of a given stock level and demand rate on the availability of the aircraft fleet. The two primary performance measures used include not mission capable for supply (NMCS) aircraft, and fully maintenance capable (FMC) sorties.

The first model discussed was developed by the logistics Management Institute (LMI) in 1972. The next model, the Wartime Assessment and Requirements System (WARS) model, was developed by AFLC in 1981. The last model discussed is the Dyna-METRIC model developed by the RAND Corporation in early 1980. The LMI and WARS models have never been wholly incorporated into the Air Force management structure. However, the basics of these models are introduced in this chapter because elements of these models will be included in future AFLC developments, such as the Requirements Data Bank.

These models are all similar to the backorder centered models in that they incorporate Palm's Theorem. However, the LMI model differs from WARS and Dyna-METRIC in that LMI represents the long-range steady-state availability of the aircraft fleet. WARS and Dyna-METRIC are similar in that they both model the dynamic situation of changing rates over time. These two models are appropriate for modeling the capability of a supply system to react to a war-time environment.

LMI Availability Centered Model

The LMI model was developed for use in conjunction with the METRIC model to compute the expected backorder reduction for each recoverable component (16:8). The LMI availability centered model converts expected backorders (and expected backorder reductions), into expected NMCS aircraft (and expected NMCS reductions). In addition, the LMI model predicts an expected number of NMCS aircraft, given an initial amount of recoverable spares exist for each recoverable component (16:11-12).

Because LMI was never adopted in whole, this section will be limited to the mathematical formulation of the basic model. The initial model discussed will treat one aircraft type, multiple components per aircraft with a no cannibalization policy. (An example will be given.) A discussion on the impact of a cannibalization policy on the model will follow.

Model Assumptions. The basic LMI model assumes (16:12):

(1) An aircraft missing a recoverable component due to stock-out will be NMCS if the component would cause an NMCS condition in real life.

(2) An aircraft cannot be NMCS unless at least one unit of a NMCS-causing component is in need of repair and a spare

is not available.

(3) The failure of any single NMCS-causing component is independent of the failure of any other component, and is also independent of the operational state of the aircraft.

(4) When more than one unit of any component is installed on an aircraft, the failure of one unit is independent from failures of any of the other like units.

Model Presentation. The objective of the LMI model is to minimize the number of NMCS aircraft given a constraining budget value. The probability that the average aircraft is missing a part is the number of backorders for that item (B_i) divided by the fleet size (F), or B_i/F. The probability that the average aircraft is missing item (i) at a random point in time is the expected backorder divided by fleet size, or $E(B_i)/F$. Therefore, the probability the average aircraft is not missing item (i) is:

$$1 - \frac{E(B_i)}{F}$$

If the quantity per aircraft of a particular item (QPA_i) is greater than one, then the expression becomes:

$$1 - \left(\frac{E(B_i)}{F*QPA_i}\right)^{QPA_i} \qquad (5.1)$$

The probability that the average aircraft is not missing any items is the product of all the probabilities of the average aircraft not missing item (i), or (16:51):

$$p(a) = \left[1 - \left(\frac{E(B_1)}{F*QPA_1} \right)^{QPA_1} \right] * \left[1 - \left(\frac{E(B_2)}{F*QPA_2} \right)^{QPA_2} \right] \ldots$$

$$* \left[1 - \left(\frac{E(B_n)}{F*QPA_n} \right)^{QPA_n} \right]$$

or

$$\prod_{i=1}^{n} \left[1 - \left(\frac{E(B_{i,n})}{F*QPA_i} \right)^{QPA_i} \right] \tag{5.2}$$

As with the backorder centered models, $E(B_i)$ is defined as:

$$E(B_i) = \sum_{X=S+1}^{\infty} (X-S)\ p(X;\lambda T) \tag{5.3}$$

<u>Example.</u> Recall the example given in the base stockage model in the previous chapter (page 47). Though this example only has one base and no depot, its simplicity allows for a ready explanation to how LMI works. If the actual stock level, expected backorders and QPA for three items is:

Item	Stock Level	E(B)	QPA
one	0	2	2
two	1	2.049	1
three	2	2.109	1

and the fleet size is six aircraft, then the long range probability of the average aircraft not missing any items (FMC rate) is:

$$p(a) = \left[1 - \left(\frac{2}{6*2}\right)^2\right]\left[1 - \left(\frac{2.049}{6*1}\right)^1\right]\left[1 - \left(\frac{2.109}{6*1}\right)^1\right] = .2965$$

The long-range NMCS rate is computed as 1 - p(a) or .7035.

As with the METRIC model, marginal analysis is used to compute which item to choose. With LMI, the objective is to pick the next item that improves the FMC rate the most. For example, using the previous example, we start with a stock level of zero for all three items to compute a FMC rate, or:

$$p(a) = \left[1 - \left(\frac{2}{6*2}\right)^2\right]\left(1 - \frac{3}{6}\right)\left(1 - \frac{4}{6}\right) = .1157$$

If one unit of item one is added, the FMC rate is .1366. If one unit of item two is added instead, then the FMC rate would be .1524. If one unit of item three was added instead, the FMC rate would be .1725. Since the best FMC rate is the result of adding one unit of item three, that item is picked first. The same process is repeated until the final constraining budget value is reached.

<u>Further Model Development.</u> LMI also allows for a full cannibalization policy where the cumulative total of missing recoverable components (due to stock-out) can be concentrated into a minimum of aircraft. The net effect of this policy is to increase the FMC rate.

Appendix B defines operational rate (OR) as the probability that at any point in time there will be no backorders, or the probability that all aircraft in the fleet (F) are available. Mathematically, it is defined as the product of ready rates (RR):

$$OR = \prod_{i=1}^{n} RR_i = p(a = F)$$

where RR is:

$$RR_i = \sum_{X=0}^{S} p(X:\lambda T)$$

With a full cannibalization policy, we can define an operational rate as a function of the number of airplanes (M) used as a source for supply (cannibalization). This has the effect of making more spare parts available, thus raising the OR rate. Mathematically, this is:

$$P(a \geq F - M) = \prod_{i=1}^{n} \left(\sum_{X=0}^{S_i + (M*QPA_i)} p(X:\lambda T) \right)$$

Next, the probability of expected number of NMCS aircraft is solved with the equation (9:12-15):

$$\text{Expected NMCS} = \sum_{M=0}^{F} \left[1 - \left(\prod_{i=1}^{n} \left(\sum_{X=0}^{S_i + (M*QPA_i)} p(X:\lambda_i T_i) \right) \right) \right]$$

Which can be generalized to:

$$E(a) = \sum_{a=0}^{F} a(P(a)) \qquad (5.4)$$

Summary. The LMI availability centered model was designed to be integrated with METRIC to provide operational measures of fleet availability. In the full scale development, the model allowed the differentiation of the impact of partial or non-NMCS broken recoverable components on fleet availability. The model was also designed to be used among many aircraft types. The LMI model was never wholly adopted though, probably due to the fact that LMI modeled steady-state situations only. For war planning purposes, LMI was inadequate.

Wartime Assessment and Requirements System (WARS)

This model was prepared by an AFLC working group in 1981. Their intent was to design a system which determined the number of recoverable components necessary to support a war scenario, and to quantify the impact of available assets on the number of aircraft available to fly the sorties required (28:1). The following section will present a simplified example of the model.

Model Presentation. The WARS model is a dynamic, probabilistic model that measures recoverable components required when transiting from peace-time to war-time. Essential to the model is the assumption that war-time daily demand rates can adequately be estimated. The peace-time requirement is calculated using a modified pipeline formula described in chapter three, where the average stock (S) required is the sum of the quantities in base repair, depot repair, and transportation in-between, or (28:13-15):

$$S = DDR[(PBR * BRCT) + (NRTS * DRCT) + (NRTS * OST)] \quad (5.5)$$

where

DDR = Daily demand rate

PBR = Percentage base repair

NRTS = Not repairable this station

BRCT = Base Repair Cycle Time

DRCT = Depot Repair Cycle Time

OST = Order and Ship Time

If the variables are all known, then a peace-time steady-state pipeline exists. If a war-time demand rate is used, then a war-time steady-state solution is reached. In this simplified example, only the demand rate differs, while all other variables are held constant. During the transition period, both the peace-time and war-time demand rates are weighted according to the point in time. For example, if the DRCT is 50 days, and the war is at the 10 day point, then the quantity in the depot repair cycle would consist of 40 days of peace-time and 10 days of war-time quantities. Mathematically, this would be (28:13-16):

$$\text{DRCT Quantity} = (DDR_P * NRTS * 40 \text{ days}) + (DDR_W * NRTS * 10 \text{ days})$$

The base assets would be computed in a similar manner. If base repair time is 5 days, and the war is at the day 2 point, then (28:13-16):

$$\text{BRCT Quantity} = (DDR_P * PBR * 3 \text{ days}) + (DDR_W * PBR * 2 \text{ days})$$

The order and ship-time quantity is computed based on the DRCT. If the DRCT is 50 days, then the units in the OST pipeline will remain at peace-time rates until day 51, when the first units repaired at war-time rates will start appearing in the OST pipeline. Therefore, at day 60 of the war, 5 days would be computed at peace-time rates and 10 days computed at war-time rates, or (28:13-16):

$$\text{OST Quantity} = (DDR_P * NRTS * 5 \text{ days}) + (DDR_W * NRTS * 10 \text{ days})$$

<u>Example.</u> The quantities required at days 10, 30, and 45 of a war are computed given the following information:

	Peace-time	War-time
DDR	2	4
DRCT	30	30
BRCT	5	5
OST	15	15
PBR	.5	.5
NRTS	.5	.5

At day 10, assets required (S) are:

$$S = [(2 * .5 * 20) + (4 * .5 * 10)] + (4 * .5 * 5) + (4 * .5 * 15) = 65 \text{ units}$$

At day 30, assets required are:

$$S = (4 * .5 * 30) + (4 * .5 * 5)$$
$$+ (2 * .5 * 15) = 85 \text{ units}$$

At day 45, the assets will reach a war-time steady-state solution as the OST pipeline is filled at war-time DDR:

$$S = (4 * .5 * 30) + (4 * .5 * 5)$$
$$+ (4 * .5 * 15) = 100 \text{ units}$$

Full Model Potential. The previous example is extremely simplified. The complete WARS model was intended to allow (28:16-17):

(1) Consideration for condemnation.

(2) Indenture of sub-components.

(3) Capability to interrupt transportation of spares due to war-time conditions.

(4) Adjustment for other variables, such as order and ship time.

Dyna-METRIC Model

There are two main deficiencies with all previous inventory models for recoverable components. First, the use of expected backorders as a performance measure did not adequately address how it affected the operational status of the aircraft fleet. In short, models using the backorder centered criteria would be difficult to use to predict combat capability. The second shortcoming of previous models is that all (except for WARS) modeled a steady-state environment. As a result, the models would be of little use in a war-time scenario where changing demand rates, repair functions, deployments and other war-time factors would have a dramatic affect on the inventory system. Muckstadt demonstrated that steady-state models seriously under-estimate stockage requirements and supply system performance in a dynamic environment (31:1).

Because of these shortcomings, Dyna-METRIC was developed to provide a dynamic model that uses operational criteria as a performance measure. The model can be characterized as multi-echelon (to include an intermediate repair level), multi-indenture, multi-item, multi-location and stochastic. Unlike the other recoverable models discussed, Dyna-METRIC has been fully incorporated into the AFLC management structure. This section on Dyna-METRIC will address the assessment portion only. Chapter six (Forecasting) will address the requirements mode.

Dyna-METRIC is a RAND developed model where elements of the METRIC, MOD-METRIC, and the LMI models have been incorporated. Dyna-METRIC took the basic idea behind METRIC, which assumed a steady-state situation, and then derived similar results for a time varying service and demand process (23:5). Hillsted and Carillo demonstrated in 1980 that Palm's theorem, which served the steady-state models, could be generalized to a dynamic process as well (see Appendix A).

Basically, Dyna-METRIC allows a manager the ability to look at a war scenario and determine the shortfalls caused by inadequate logistical support. For the first time, he is able to predict readiness of aircraft squadrons as determined by the amount of logistics resources (22:2). Specifically, Dyna-METRIC provides (35:3-7):

(1) The operational performance measures of aircraft availability and FMC sorties flown. Dyna-METRIC arrives at these results by using detailed resource counts and process delay times to forecast combat capability.

(2) The effects of disruptions in the supply system caused by a shift from a peace-time to war-time environment. For example, a deployment of an aircraft squadron would mean an interruption in the supply pipeline until the squadron was in place. Dyna-METRIC allows for these interruptions.

(3) A capability to measure the effects of repair constraints and priority repair management.

(4) The ability to detect problem component items ranked in order of the highest probability that a component caused the most grounded aircraft.

(5) An analytical tool which determines alternate cost-effective repair or stock purchases that would achieve a target performance goal given a war-time scenario.

Model Limitations. Version 3.4 of Dyna-METRIC operates under the following assumptions (35:32):

(1) Repair procedures and productivity are unconstrained and stationary (except for test-stand simulation).

(2) FMC sortie rates do not directly reflect flight-

line resources and the daily employment plan.

(3) Component failure rates vary only with user-requested flying intensity.

(4) Aircraft at each base are assumed to be nearly interchangeable, (i.e., single mission design).

(5) Repair decisions and actions occur when testing is complete. (Version 4 allows for NRTS before testing a failed item.)

(6) Component failure rates are not adjusted to reflect previous FMC sorties accomplished.

(7) All component repair processes are identical at all echelons.

(8) No lateral resupply allowed.

Version 4 further overcomes some of these limitations. In particular, repair constraints can be modeled and demand rates can vary depending on the location of the base. Additionally, failures can occur as a result of flying hours or numbers of sorties flown (24:265).

Model Description. This sub-section will describe version 3.4 of Dyna-METRIC as currently used in AFLC. However, since version 4 is expected to be introduced in the near future, some of its major changes will also be addressed. Appendix C contains an outline of the computer algorithm and formulas currently used in Dyna-METRIC.

The primary objective of Dyna-METRIC is to avoid degradation of aircraft mission capability due to shortages of recoverable components. To achieve this goal, the local supply of these components needs to exceed the number of components tied up in the repair and resupply pipelines (22:3). The basic concept underlying Dyna-METRIC is to view an aircraft as a collection of components, each with its own particular failure rate and repair cycle time. Dyna-METRIC forecasts the quantity of each component in the repair and resupply pipelines based on the component's interaction with the operational war-time demand. These pipeline quantities are combined and the effect on aircraft availability and sortie rate is estimated using statistical methods (35:8).

Computation of pipeline quantities is central to Dyna-METRIC. The repair and resupply pipeline is similar to the system described in Chapter Three, pages 40-42. One major difference is that Dyna-METRIC adds a Centralized Intermediate Repair Facility (CIRF) to the system. Each of the repair stations or resupply channels represents a pipeline in the model. If there are not enough components to cover each of these pipelines, then the result will be shortages (or "holes") of the components on the aircraft resulting in backorders. These backorders may or may not ground the aircraft depending on the essentiality of the components (22:3). Dyna-METRIC computes the total availability of resources and then translates the information into sortie capability and NMCS aircraft.

Dyna-METRIC is a multi-indenture model that considers the impact of subcomponets (SRUs) on LRUs. Version 4 also considers a third level of indentured parts called sub-SRUs.

The input to the computer model identifies the indenture relationship between LRUs, SRUs and sub-SRUs. Dyna-METRIC computes expected pipeline quantities for each LRU, SRU and sub-SRU at the base, CIRF and depot levels. The model uses a building block approach to determine the overall LRU pipeline. A pipeline for a SRU awaiting parts (AWP) is computed for the sub-SRU in stock and in repair. The same holds true for an LRU AWP (24:10). In this manner, the total pipeline quantity can be determined.

The key equation to Dyna-METRIC is the determination of the expected number of LRUs in the pipeline. This quantity is a function of the demands per day (λ) and repair process (T). The demand function for an item d(s), which represents λ, is a function of the following inputs:

(1) Failures per flying hour (failure rate).

(2) Flying hours per sortie at time "t".

(3) Quantity of an item per aircraft (QPA).

(4) Sorties per aircraft at time "t".

(5) Aircraft fleet size at time "t".

(6) Percentage of aircraft at a particular base with the component installed (application factor).

The mathematical relationship is defined as the product of these factors, or:

$$d(s) = \frac{\text{failures}}{\text{flying hr}} * \frac{\text{flying hrs @ t}}{\text{sortie}} * (\text{QPA}) * \frac{\text{sorties @ t}}{\text{acft}} * (\text{acft @ t}) * \begin{matrix}\text{Application}\\ \text{Factor}\end{matrix} \qquad (5.6)$$

These variables are all subject to change. Flying hours per sortie can change as missions change. Number of sorties per aircraft can change as a result of changes in flying rates (22:9). Failure rates per component can also change with a change from peace-time to war-time. This new failure rate is computed as a nonlinearity factor where 1.0 denotes no change and 2.0 would represent a doubling of the failure rate (35:18). This new rate represents a component that would be used more (or even less) in war-time, such as gun barrels. In addition, the fleet size can change due to attrition, which is computed as:

Attr day N = (# non-attr acft day N) * (sortie rate)
* (Attr factor)

Version 4 will also allow for an on-shore and off-shore demand rate for LRUs (24:171). This distinction accommodates a number of plausible reasons for a change in demand rates between bases. For example, a change to a sub-arctic environment would increase the demand rate for systems using hydraulic hose components. Another variation that version 4 allows is the option of entering a war-time sustained demand rate at any point in the war scenario. This allows for a more accurate representation of when an aircraft squadron (and demand rates for the LRUs) would actually begin flying at a war-time rate (24:139).

The repair process F(t,s), which represents T, is computed as a probability function of a component entering repair. It is defined as (22:9):

$$F(t,s) = \text{prob (component entering repair at } s \text{ is still in repair at } t.)$$

$$= \text{prob (Repair time} > t - s \text{ when started at } s.)$$

This function can either be computed deterministically or as an exponential distribution. The Dyna-METRIC package at AFLC uses the exponentially distributed repair times where:

$$F(t,s) = \left\{ \begin{array}{ll} 1 & \text{if } t < Ra \\ e^{-\frac{t-Ra}{RCT}} & \text{if } s < Ra \leq t \\ e^{-\frac{t-s}{RCT}} & \text{if } Ra \leq s < t \end{array} \right\} = E(T_t) \qquad (5.7)$$

where

Ra = Repair availability (Repair assets are in place.)

RCT = Repair cycle time

The expected pipeline quantity can now be computed using Palm's theorem. The demand function multiplied by the repair function equals the expected pipeline quantity, or:

$$U(t) = \sum_{s=0}^{t} d(s) \times F(t,s) \qquad (5.8)$$

where the expected number in the pipeline on day t equals those demands that are in repair now at time t. The expected pipeline quantities can be broken into local and off-base segments by factoring in the PBR and the NRTS percentage. This allows for dissimilar demand rates and repair functions.

The two results can then be summed together for an average total quantity of components in the pipeline (22:14).

To compute the probability of various pipeline sizes, the poisson distribution is used to compute the following equation (22:21):

$$p(\# \text{ in pipeline} \leq k \text{ at time } t) = \sum_{X=0}^{k} \frac{U(t)^{X} e^{-U(t)}}{X!} \qquad (5.9)$$

For example, to compute the pipeline size for local repair only, the U(t) used in the equation would be computed with PBR factored in equation (7). Figure 9 gives a graphic description of the steps explained to this point.

The next major step in the Dyna-METRIC process is to compute the performance measures using the expected backorders to determine the number of "holes" for each component on an aircraft. The now familiar equation for expected backorders is:

$$E(B) = \sum_{X=S+1}^{\infty} (X-S)\ p(X|\lambda T) \qquad (5.10)$$

With expected backorders computed for each component, the operational performance measures can now be computed, such as the number of NMCS aircraft. (Appendix C gives the full expansion of all the following operational performance measures.) If we take the first case of a no cannibalization policy and QPA is limited to one per aircraft, the expected number of NMCS aircraft at time (t) is equal to the total

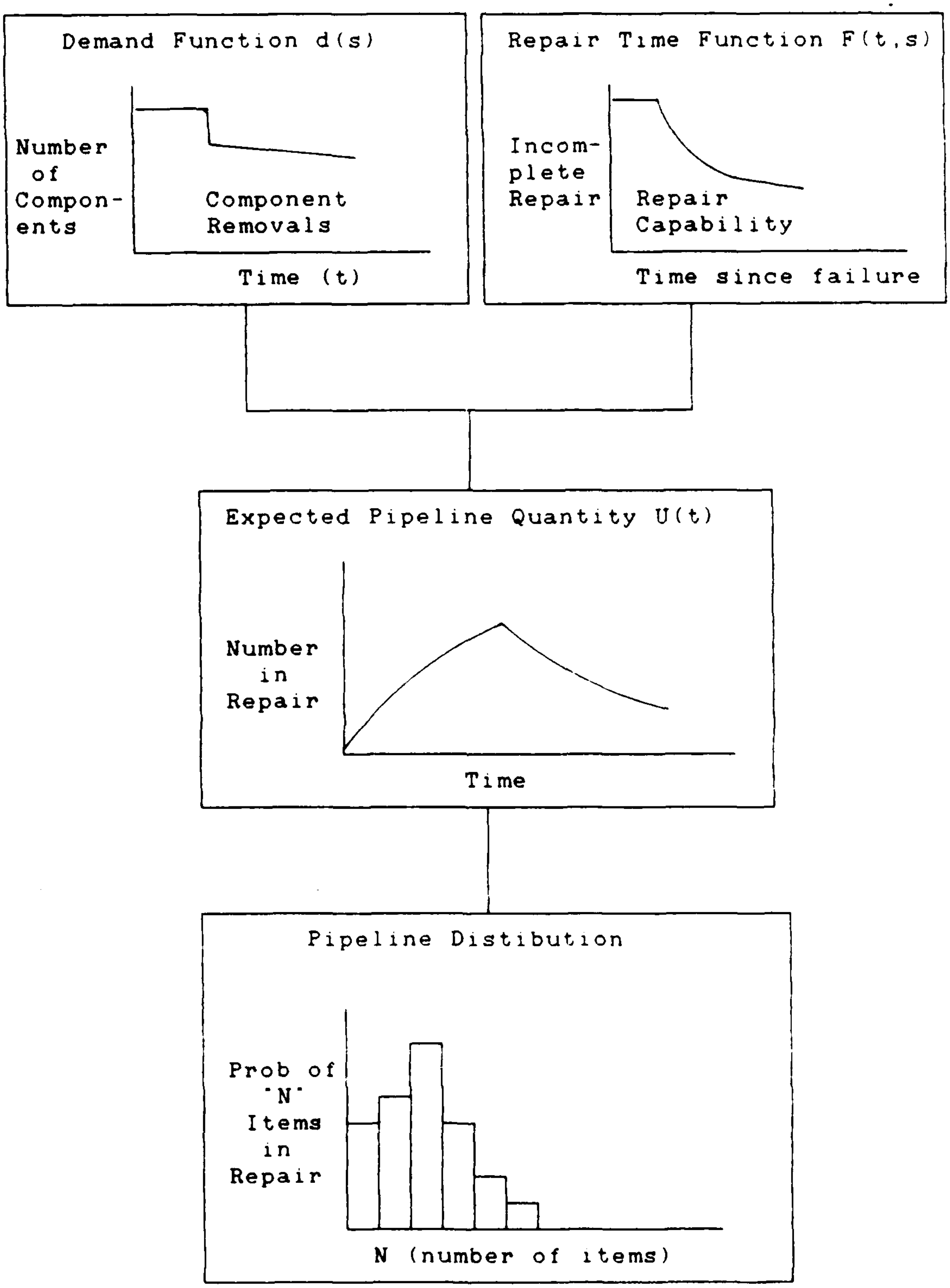


Figure 9. Computation of Pipeline Quantities and Distribution

number of aircraft items multiplied by the probability an aircraft is not missing (i) parts. The equation is (22:32):

$$E(NMCS) = NA(t)\left[1 - \prod_{i=1}^{n}\left(1 - \frac{E(B_i(t))}{NA(t)}\right)\right] \quad (5.11)$$

If QPA is more than one, then the equation is expanded to (22:32):

$$E(NMCS) = NA\left[1 - \prod_{i=1}^{N}\left(\sum_{y=0}^{NA*Q_i} \frac{\binom{Q_i*NA-y}{Q_i}}{\binom{Q_i*NA}{Q_i}} P(B_i=y)\right)\right] \quad (5.12)$$

where

Q_i = Quantity of items i per aircraft

$P(B_i = y)$ = Probability that aircraft have no shortages of i when y shortages exist

$\binom{Q_i*NA-y}{Q_i}$ = No. combinations of Q from good installed parts

$\binom{Q_i*NA}{Q_i}$ = No. combinations of Q from all installed parts

Note: The last two expressions are factorials where:

$$\binom{N}{R} = \frac{N!}{R!(N-R)!}$$

A full cannibalization policy will help to lower the expected number of NMCS aircraft by consolidating the aircraft "holes" into as few as aircraft as possible. For example, if five aircraft were missing six items (denoted by

X) for four separate type item groups as depicted in table 7, then the total number of NMCS aircraft is all five aircraft. After consolidating the "holes" into as few aircraft as possible, as depicted in table 8, then the total NMCS aircraft is reduced to two. The mathematical equation for the expected number of NMCS aircraft at time (t) for a full cannibalization policy with the possibility of QPA greater than one is:

$$E(NMCS) = \sum_{j=0}^{NA-1} \left[1 - \prod_{i=1}^{N} \left(\sum_{k=0}^{S_i + jQ} \left(\frac{U_i^{K} e^{-U_i}}{K!} \right) \right) \right] \quad (5.13)$$

where

U_i = Number of LRUs in the pipeline

K = Stock level

In reality, a no cannibalization or full cannibalization policy represent the extreme cases. Some LRUs are readily cannibalized while some are not (due to inaccessibility, removal time, etc.). Version 4 corrects this shortcoming by assigning a code to each LRU denoting the feasibility of cannibalization.

Thus Dyna-METRIC computes at time (t) the number of NMCS aircraft out of the total fleet after attrition. This performance measure is then used to determine the expected number of sorties that can be generated that day. Appendix C shows the computation for this value. Dyna-METRIC will also compute a problem list of LRUs starting with the LRU that has

Table 7. LRU Shortages per Aircraft

Item #	Aircraft # 1	2	3	4	5
A	X			X	
B		X			
C			X		
D			X		X

Table 8. LRU Shortages per Aircraft after Consolidation

Item #	Aircraft # 1	2	3	4	5
A	X		X		
B	X				
C			X		
D	X		X		

the highest probability of grounding the most aircraft. The problem components are rank-ordered by a factor W(i) where (24:101):

$$W(i) = \sum_{b=1}^{B} \left[\left(\frac{E(B_i)}{QPA_i} \right) \left(\frac{1}{\text{Application fraction of LRUs at base b}} \right) \right] \quad (5.14)$$

<u>Example.</u> To demonstrate how Dyna-METRIC works, a simple problem with three components, single indenture, local repair only (PBR = 1), and a no cannibalization policy will be computed to arrive at the expected number of NMCS aircraft. To simplify the problem, only the first LRU's expected backorder will be computed. The other two LRU's expected backorders will be 0.3 and 1.0. The following information is given for item one:

Failure/flying hour	=	.01 failures/hour
Flying hour per sortie @ t	=	2.5 hours/sortie
Quantity per aircraft	=	1.0
Sorties per aircraft @ t	=	3.0 sorties/aircraft
Number of aircraft (NA)	=	10.0 aircraft
Repair Cycle Time (RCT)	=	5.0 days
Percentage factor	=	1.0
Start of repair time (s)	=	day 5
Repair availability (RA)	=	day 8
Time (t)	=	day 10
Total units in System (S)	=	2.0 units in stock

First, the demand function, d(s) is computed using Eq (5.6) to arrive at a DDR:

$$d(s) = (.01)*(2.5)*(1)*(3)*(10)*(1.0) = .75 \text{ failures}$$

The repair function, F(t,s) is computed using Eq (5.7) and multiplied by d(s) for each day. The results are then summed to find U(t) using Eq (5.8):

Day	F(t,s)	d(s)	F(t,s)*d(s)
5	1	.75	.75
6	1	.75	.75
7	1	.75	.75
8	1	.75	.75
9	.82	.75	.615
10	.67	.75	.502

Expected units in pipeline = U(t) = 4.117 ≈ 4.1

We next compute the expected backorders using Eq (5.10). The entering arguments are U(t) = λT = 4.1 and S = 2. The computed E(B) is 2.1975. If using a no cannibalization policy with QPA = 1 per aircraft, Eq (5.11) is computed with the resulting expected number of 3.188 NMCS aircraft. Thus in our simple example, we would expect to only have 6.182 aircraft available at day 10 with the rest of the fleet grounded due to backorders for the three components.

<u>Summary.</u> By generalizing the simple example to the vast capabilities of the Dyna-METRIC model, one can easily see how Dyna-METRIC can become an essential assessment tool in

determining war-time capabilities in terms of stockage of recoverable components. Because Dyna-METRIC allows "what if" analysis, a planner is able to optimize the war-time capability by manipulating the control variables. Of all the dynamic models addressed in this chapter, Dyna-METRIC comes closest to modeling the real world.

VI. Forecasting

Overview

The one common element found in any inventory system is some type of forecasting technique for determinating future demand. Several factors determine the choice of a forecasting method. Some of the more important considerations include (21:112,119):

(1) The time length of the required forecast.

(2) The level of technical sophistication of the people using the system.

(3) The cost of forecasting systems depending on computer, manpower, and time requirements.

(4) The currency and accuracy of the available data.

(5) The importance of the level of accuracy of the forecast.

Air Force inventory systems have many diverse characteristics that call for different forecasting methods. On one hand, the Air Force manages thousands of low-cost, non-recoverable items that are acquired through Economic Order Quantity (EOQ) type replenishment systems. On the other hand, the Air Force manages high-cost reparable assets that use pipeline-type inventory systems. A further complication is that the Air Force logistician must manage these supply systems not only in peace-time, but also transition to and support a war-time scenario.

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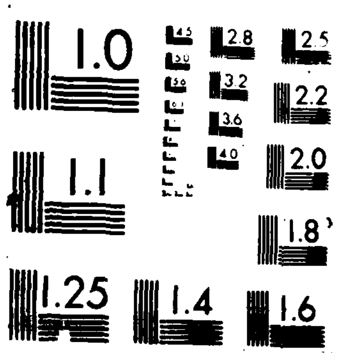

1.0
2.8
2.5
3.2
2.2
3.6
4.0
2.0
1.1
1.8
1.25
1.4
1.6

This section will first address time series models used in forecasting Air Force low-cost non-recoverable assets. The methods addressed include moving average and exponential smoothing models. Second, a sub-section on a regression technique is included to compare and contrast time series models with econometric type models. Third, this chapter addresses the different forecasting methods used to determine demand rates at base and depot level for all assets.

Time Series Methods

Time series model forecasts depend solely on the values of previous data. They assume external factors generally do not affect the forecast.

Moving Average Methods. These methods are the simplest to use for forecasting, and their advantages are substantial. For example, the time horizon for forecasting can be applied to short as well as long range forecasts. Further, the method is easily understood and the forecasts can be updated rapidly. They also accomodate fluctuations, to a degree, with appropriate averaging periods. If dealing with stable data, accuracy can be high.

There are, however, drawbacks to the moving average methods. For example, past history of data is essential. If many periods are required for averaging, then computer requirements can be substantial. In unstable situations, forecasts can be inaccurate. In addition, these methods do not anticipate turning points in the data.

The easiest of the two methods addressed is the simple moving average method which is used to average-out any random movement in the data over a specified period of time. Mathematically, this is represented as:

$$F_t = \sum_{i=1}^{n} \frac{A_{t-i}}{n} \qquad (6.1)$$

where

F_t = Forecast for time t

A_t = Actual data for time t-i

n = number of periods used for averaging

<u>Example.</u> If given the following time series data:

Period	Actual Data	Period	Actual Data
1	14	6	34
2	19	7	36
3	20	8	45
4	22	9	43
5	28	10	39

A forecast for the eleventh period using a four period moving average would be:

$$\frac{36 + 45 + 43 + 39}{4} = 40.75$$

One problem with the simple moving average method is that older data have equal impact on the forecast with recent data. If more importance is to be placed on the latest data, a weighting factor can be added to the actual data points to arrive at a weighted moving average. Mathematically, this is:

$$F_t = \sum_{i=1}^{n} W_i (A_{t-i}) \qquad (6.2)$$

where W_i is the weight factor. (The sum of all W_i must equal one.)

<u>Example.</u> If we used the data set from the previous example, and assigned a weight factor of .4 for the most recent data point and .3, .2 and .1 for the trailing data points, the forecast for the eleventh period would be:

$$.1(36) + .2(45) + .3(43) + .4(39) = 41.1$$

<u>Expontential Smoothing Method</u>. The simple exponential smoothing method assigns exponentially declining weights to current and previous values, by using a single weight (α) called the expontial smoothing constant. Mathematically, this is represented as (39:98):

$$F_t = \alpha(A_{t-1}) + (1-\alpha)^1(A_{t-2}) + (1-\alpha)^2(A_{t-3}) \ldots + (1-\alpha)^n(A_{t-n})$$

which is mathematically the same as:

$$F_t = \alpha(A_{t-1}) + (1-\alpha)F_{t-1} \qquad (6.3)$$

where

F_t = Forecast value

A_t = Actual data value

α = smoothing constant (must be between 0 and 1)

The chief advantage of this method is that it incorporates all past data from the initiation of the technique. However, to compute a forecast, only the last period's actual and forecast value need be known. Therefore, the formula is computationally convenient, and since so few data points need be stored, computer resources are more effectively used. The forecaster can determine the weight of previous data by assigning a value to α. If more weight on the latest data is deemed necessary, then a larger α is assigned. If more smoothing of all the previous data is desired, then a smaller α is required.

Example. If we used the data set from the first example, assigned an α value of .3, and arbitrarily assign the forecast for the first period equal to the actual data value for that period, the result would be:

Period	Actual	Forecast	Period	Actual	Forecast
1	14	14	6	34	21.3
2	19	14	7	36	25.1
3	20	15.5	8	45	28.4
4	22	16.9	9	43	33.4
5	28	18.4	10	39	36.3
			11	...	37.1

Regression Techniques

These techniques establish a relationship between a dependent variable, on which future values will be forecast, and the influence of independent variables (20:78). The chief characteristic of this method is that it is a causal model. In other words, it implies that the outcome is dependent on underlying factors.

Simple linear regression analysis involves only one independent variable and is mathematically represented as a line:

$$Y = a + bX \tag{6.4}$$

where

Y = Dependent variable

a = Y axis intercept

b = Slope of the line

X = Independent variable

Multiple regression involves several independent variables and is represented as:

$$Y = a + b_1X_1 + b_2X_2 \ldots + b_nX_n \tag{6.5}$$

Multiple regression analysis requires the use of a computer, while linear regression does not (though a computer does greatly simplify the process). This section will address the basic linear regression equation only, though the concepts remain the same for multiple regression. Hypothesis testing or confidence intervals are not discussed.

The basic idea behind linear regression is to solve a trend line, so that the sum of the deviations between the plot of data and the trend line is minimized (see figure 10). The following equations determine the values of the slope and Y-intercept of the fitted trend line (29:401-403):

$$\text{Slope (b)} = \frac{SS_{xy}}{SS_{xx}} \tag{6.6}$$

$$\text{Y-intercept (a)} = \bar{Y} - b\bar{X} \tag{6.7}$$

where

$$SS_{XY} = \sum_{i=1}^{n} X_i Y_i - \frac{\sum_{i=1}^{n} X_i \quad \sum_{i=1}^{n} Y_i}{n} \tag{6.8}$$

$$SS_{XX} = \sum_{i=1}^{n} X_i^2 - \frac{\left(\sum_{i=1}^{n} X_i\right)^2}{n} \tag{6.9}$$

where n = sample size

In addition, a coefficient of determination (r^2) can be computed which measures the percent of variation in Y that is explained by X. This is computed as (29:421-422):

$$r^2 = 1 - \frac{SSE}{SS_{YY}} \tag{6.10}$$

where SSE represents the sum of squared errors and SS_{YY} is the sum of the squared independent variables.

Mathematically, these are:

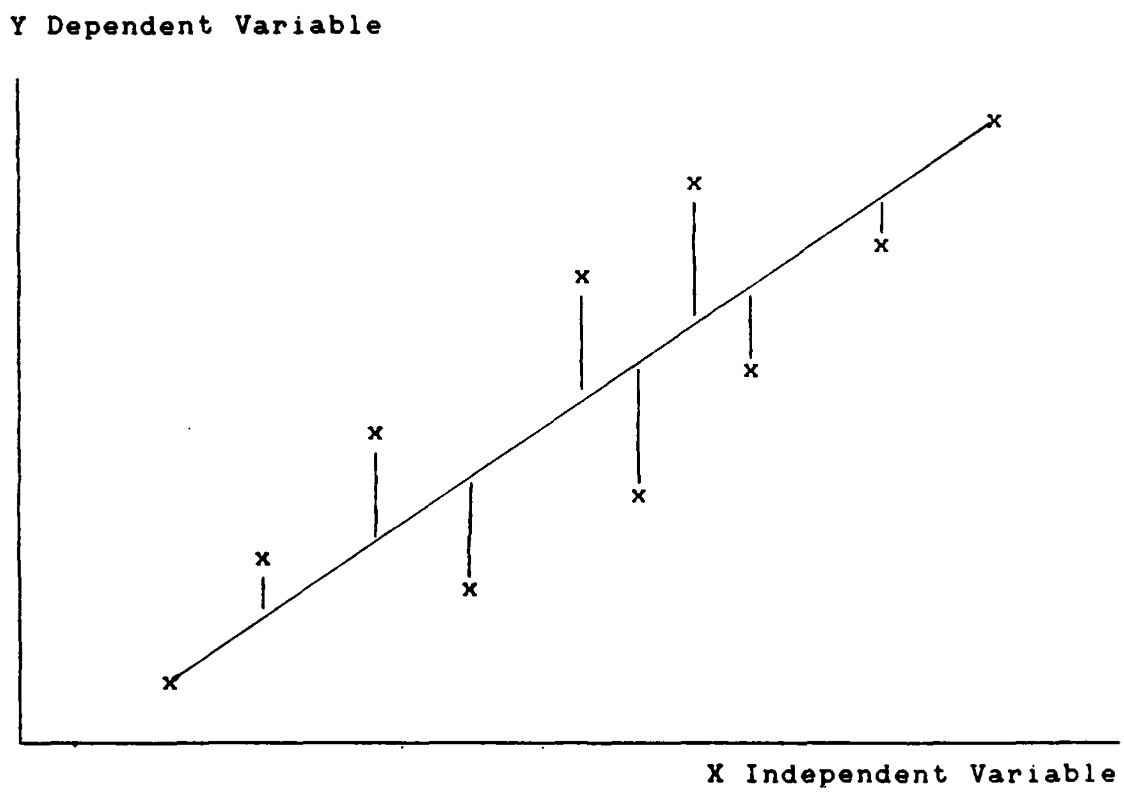


Figure 10. Fit of Least Squares Line

$$SSE = SS_{YY} - b(SS_{XY}) \tag{6.11}$$

$$SS_{YY} = \sum_{i=1}^{n} Y_i^2 - \frac{\left(\sum_{i=1}^{n} Y_i\right)^2}{n} \tag{6.12}$$

The square root of (r^2) is the Pearson product moment of correlation and is the strength of the linear relationship between variables X and Y. This number always falls between negative and positive one. A zero implies little or no correlation while a positive or negative one implies that all points fall on the fitted trend line. A positive value means that values of Y increase with values of X while a negative value implies the reverse.

Thus with a solved equation for the fitted trend line, forecasts can be made by substituting in the values for the independent variable and solving for the dependent variable.

Example. If given the following data set, solve for the trend line equation and forecast for the sixth period if the estimated value for X in the sixth period is 5.

Period	X	Y
1	6	20
2	6	18
3	4	10
4	2	6
5	3	11

The first step is to compute the following summations:

	X	Y	X^2	XY	Y^2
	6	20	36	120	400
	6	18	36	108	324
	4	10	16	40	100
	2	6	4	12	36
	3	11	9	33	121
Totals:	21	65	101	313	981

Next, using Eqs (6.6, 6.7, 6.8, and 6,9), the SS_{XY}, SS_{XX}, the slope and the Y-intersection are computed resulting in:

$SS_{XY} = 40$ Slope (b) = 3.125

$SS_{XX} = 12.8$ Y intersection (a) = -.125

The trend line equation is thus:

$$E(Y) = -.125 + 3.125(X)$$

By substituting in the estimated X value of 5 into the equation, the forecast for the sixth period is an expected value of 15.5. To compute the coefficient of determation, Eqs (6.11 and 6.12) are computed, resulting in $SS_{YY} = 136$ and SSE = 11. The r^2 is then computed as .919 using Eq (6.10). The Pearson correlation product moment (r) would be .9587.

The major advantages of regression techniques over the time series models are (7:23-24):

(1) Regression models seek underlying factors for the dependent variables.

(2) The reliability of the forecast can be measured in objective probablistic terms.

The disadvantages are:

(1) Regression requires a large amount of data resulting in high cost and time.

(2) The causal relationship in the variables need to be monitored for any changes.

(3) Forecasting outside of the range of the variables is suspect.

Base Level Forecasting

In chapter two, the EOQ was computed (for non-local purchase) as:

$$EOQ = \frac{8.3 \sqrt{DDR(365)\ (\text{Unit Price})}}{\text{Unit Price}}$$

where DDR represents the daily demand rate. The DDR is the forecasting element of the equation, and is derived from a modified exponential smoothing model with a variable smoothing parameter. The formula for calculating DDR is:

$$DDR = \frac{CRD}{MAX[180, \text{ or } (\text{current date} - DOFD)]} \quad (6.13)$$

where

CRD = Cumulative Recurring Demand

DOFD = Date of First Demand

The minimum 180 days is established to prevent new stock items from being overstocked. Every six months, the CRD and DOFD is adjusted to reflect the most recent usage data. The adjustments are:

$$CRD = DDR * MIN[365, \text{ or } (\text{current date} - DOFD)] \quad (6.14)$$

$$DOFD = MAX[DOFD, \text{ or } (\text{current date} - 365)] \quad (6.15)$$

The result of these adjustments is to add a variable smoothing parameter. During the six month interval, when a demand is made for a particular item, the DDR is revised. The net effect of the six month adjustment is to insure that the DDR is based on no more than 545 days of demand history. New demands enter the forecast with a smoothing constant (α) where (33:3):

$$\alpha = \frac{n}{365 + n} \quad (6.16)$$

and n is the number of days since the last adjustment.

<u>Example.</u> Given the CRD for an item is 190, current Julian date of 7015, and a DOFD of 6001, the DDR is computed as 190/380 = .5. If by Julian date 7180, the CRD has increased to 285, the DDR would be recomputed as 285/545 = .5. However, on this date the DOFD and CRD are readjusted so that:

DOFD = MAX[6001, (7180 - 365)] = 6180

CRD = .5 * MIN[365, (7180 -6180)] = 180

With a readjusted CRD of 180, the new DDR is recomputed as DDR = 180/365 = .49.

Depot Level Forecasting

Forecasting for Non-Recoverable Assets (D062). In chapter two, the AFLC EOQ formula was given where:

$$EOQ = \sqrt{\frac{2AC}{H}}$$

where

A = Annual demand rate

C = Cost to order

H = Cost to hold

The annual rate is derived by a simple unweighted moving average of eight quarters of demand histroy plus a fraction of the current quarter. The current quarter ratio is computed as (3:78):

$$\frac{\text{(Current date - last shift date)}}{91} \qquad (6.17)$$

If there are two or more quarters of demand history on an item, a monthly demand rate (MDR) is computed as (3:78):

$$MDR = \frac{\text{DUC for current \& previous Qtrs (up to 8)}}{3 * \text{(Qtr + current Qtr tally)}} \qquad (6.18)$$

where DUC is the Demands Used in Computation. If there is less than two quarters of demand history, MDR is computed as MDR = DUC/6. The MDR is converted to a program monthly demand rate (PMDR) which is the MDR multiplied by a peacetime program ratio (PPR) (3:78). The program annual rate (PAR) that is used in the EOQ formula is simply the PMDR multiplied by 12.

For example, given eight quarters of demand history with a total DUC of 400, the MDR is computed as 400/(3*8) = 16.6. If we assume a PPR of 1.0, yielding a PMDR of 16.6, the PAR can then be computed as 12*16.6 = 199.2. This means AFLC expects to use at least 199 of these assets in the coming year, based on historical trend data.

Forecasting for Recoverable Assets (D041). AFLC manages recoverable assets (Expendability-Recoverablity-Reparability-Category (ERRC) designator XD) through the D041 requirements system. They assume these assets are normally repaired at base and depot level. Of prime importance to the system is the generation of reparable assets at both base and depot level.

The D041 forecasting system uses a 24 month non-weighted moving average to capture the information from the previous eight quarters. This eight quarter usage history is updated each quarter by adding the current quarter's usuage and dropping the oldest quarter of usage (2:9.1). The most important factor computed for each quarter is the Total Organizational and Intermediate Maintenance Demand Rate

(TOIMDR). The TOIMDR indicates the total demand rate likely to occur during operational use of an aircraft or system (2:9.12). The rate is computed as: (number of base failures repaired on base + number of failures sent off base for repair + base condemnations) divided by the base period past installed program (2:9.11)

The D041 system maintains three fiscal year forecasts of the TOIMDR based on a moving average. The forecast is important because buy quantities and repair requirements are based on this system. However, the equipment specialist (ES) in charge of the particular item has a lot of influence in the final forecast. AFLCR 57-4 allows the ES to override the system and input estimated rates in order to show the most accurate requirements that are possible (2:6.6). In short, the D041 forecast system combines a quantitative approach (non-weighted moving average), with qualitative input to forecast requirements.

Dyna-METRIC. In addition to the assessment mode addressed in chapter five, Dyna-METRIC has a requirements mode as well. At present, war reserve materiel (WRM) requirements are computed in the D029 system in accordance with AFLCR 57-18. However, Dyna-METRIC in the requirements mode is scheduled to replace D029 by 1992 with the addition of the Requirements and Execution Availability Logistics Module (REALM) to the Weapon System Management Information System (WSMIS).

In the requirements mode, Dyna-METRIC determines the number of spare parts required to satisfy a certain level of aircraft availability. The basic approach is to compute, for each item of interest, the marginal increase in spare parts to achieve a given capability over those already input or determined for a previous time (22:62).

The overall objective function is the minimization of the total cost of each item subject to a probability that the number of NMCS aircraft less than a certain level meets or exceeds a set confidence level. Mathematically, this is (22:63):

$$\text{Minimize} \sum_{j=1}^{m} \sum_{i=1}^{n} C_i S_{ij} \tag{6.19}$$

subject to

P(#NMCS aircraft $\leq$ Y) $\geq$ confidence level

where

C_i = Cost of units

S_{ij} = Stock for all items for all bases

Y = Target NMCS rate

The probability (# NMCS aircraft $\leq$ Y) equals:

$$\prod_{i=1}^{N} \left(\sum_{k=0}^{S_i + jQ} \frac{U_i^{K} e^{-U_i}}{K!} \right) \tag{6.20}$$

where

U_i = Number of LRUs in the pipeline

K = Stock level

which represents a full cannibalization policy (see Eq 5.13).

Once the stock level required is computed to meet an operational requirement in a war plan, then the number of items to acquire can be determined. If funding is a constraint (i.e. not all items can be purchased), then marginal analysis is used to determine which items to buy. Basically, the item that is purchased is that item that gives the largest increase in the confidence level at the lowest cost (22:64).

In summary, Dyna-METRIC allows a forecast for required WRM spares based on an actual war plan program. The importance of an acquisition program for spares is heightened as it now becomes tied to bottom line Air Force requirements for projection of airpower.

Appendix A: Palm's theorem

The poisson distribution, or one of its variations, is the most widely used distribution for steady-state and dynamic inventory models. Using the poisson distribution and Palm's theorem adds versatility to inventory models. Palm's theorem states that if demand is poisson, then the number of units in steady-state resupply is also poisson for any distribution of resupply. The poisson state probabilities depend on the mean of the resupply distribution, and not on the resupply distribution itself (18:2).

The theoretical basis for Palm's theorem depends on four assumptions (11):

(1) The demand process is poisson with a rate of units over a unit time period.

(2) The demand process is independent of the repair process.

(3) The repair process is random with a mean time of T time units.

(4) Slack service capacity exists. In other words, repair is not constrained by repair resources.

If these assumptions hold true, then the number of assets in resupply (X), is also poisson with a mean of λT. The probability of a steady-state X number of units in resupply is defined as:

$$P(X) = \frac{e^{-\lambda T}(\lambda T)^{X}}{X!} \qquad (A.1)$$

Feeney and Sherbrooke extended Palm's theorem to include the compound poisson. The compound poisson is covered more in depth in the base stockage model in Chapter Four. Due to this generalization, Palm's theorem has served as the basis not only for the base stockage model, but for the METRIC and MOD-METRIC models as well.

In 1980, Hillsted and Carillo presented the mathematical proof that generalized Palm's theorem to a nonhomogeneous poisson process for a nonstationary case. This allowed for a dynamic rather than a steady-state situation using a poisson process and serves as the basis for the Dyna-METRIC model now in use at AFLC. The assumptions for a nonhomogenous poisson process are (23:5-6):

(1) The number of demands existing at time $t = 0$.

(2) The numbers of demands in disjoint time increments are independent of each other.

(3) The probability of more than one demand in an increment becomes infinitesimally small as the increment gets small.

(4) The probability of one demand in any increment is given by an intensity function times the length of the increment as the increment gets small.

Hillsted and Carillo mathematically prove that nonstationary demands, described by the parameter $d(s)$, and nonstationary service, given by $F(s,t)$, are defined so that (23:9):

$$T = \int_0^t F(s,t)d(s)ds \qquad (A.2)$$

If we are using discrete data, then the above integral can be represented by the summation from time s to time t, or:

$$T = \sum_{s=0}^{t} F(s,t) * d(s) \qquad (A.3)$$

This formula of course is the key equation in the Dyna-METRIC model, and represents a non-homogenous (or time varying), compound poisson distribution.

Appendix B: Performance Measures

This appendix defines the performance measures used throughout this thesis. The performance measures addressed are fill rate, ready rate, expected backorder, and operational ready rate. The mathematics will accompany each definition. The assumptions for the performance measures are as follows (9:6-7):

(1) A one-to-one requisition of recoverable items exists. In other words, a demand on supply for a serviceable item is accompanied by turn-in of a reparable item.

(2) Unsatisfied demands results in backorders.

(3) Demand for an item is a function of the markov property where numbers of demands that occur in any period of time are independent of demands in any other periods.

(4) Stationarity of demand. The number of demands in a given time period is a poisson random variable whose probability distribution depends only on the length of the time period; identical time period lengths have the same probability.

(5) Resupply time and demand are independent of each other.

Fill Rate (FR). This performance measure is used widely throughout the Air Force. FR is defined as the total number of units over a fixed interval of time divided by the total demand for the units. The resulting quotient is the percentage of demands filled, or fill rate (9:2). If demand is poisson distributed, then Palm's theorem can be used to predict FR for a given stock level. Mathematically, this is:

$$FR_i = \sum_{X=0}^{S_i - 1} p(S \mid \lambda_i T_i) \qquad (B.1)$$

where

FR_i = fill rate for item i

X = number of items in resupply

S_i = stock level for item i

This equation is the same as:

$$1 - p(X \geq S \mid \lambda_i T_i)$$

or

$$FR_i = \sum_{i=S}^{\infty} p(X \mid \lambda_i T_i) \qquad (B.2)$$

Eq (B.2) allows for easy computation using the poisson cumulative tables found in appendix D. If for example, $\lambda T = 4$ and $S = 3$, then FR can be determined by entering the cumulative poisson tables and extracting .7619. The fill rate is thus (1-.7619) = .2381.

<u>Ready Rate (RR).</u> This performance measure is the probability of no backorders for each item at a random point in time. Mathematically, it is the inclusive probabilities of the number of items in resupply (X) being no greater than the stock level (S), or (11):

$$RR_i = \sum_{X=0}^{S_i} p(X \mid \lambda_i T_i) \qquad (B.3)$$

To simplify computation of RR, equation (3) is the same as:

$$RR_i = 1 - \sum_{S+1}^{\infty} p(X \mid \lambda_i T_i) \qquad (B.4)$$

If for example, $\lambda T = 4$ and $S = 3$, then we enter the cumulative poisson table with the entering arguments of $\lambda T = 4$ and $S + 1 = 4$. The result is .5665. The RR is (1 - .5665) = .4335.

Expected Backorder. This performance measure is defined as a due-out of a unit from supply (9:2). It is more versatile than FR because backorders also consider the time length of backorders while FR does not. The equation is:

$$E(B_i) = \sum_{X=S_i+1}^{\infty} (X-S_i)\, p(X \mid \lambda_i T_i) \qquad (B.5)$$

Figure 11 shows a poisson distribution with $\lambda T = 2$ and $S = 1$. Expected backorders would consist of the part of the distribution that exist from (S + 1) to infinity, or the shaded area in figure 11. To determine the expected backorder, enter the individual poisson tables with $\lambda T = 4$ and (S + 1) to infinity, or S = 2, 3, 4, 5, 6, 7, 8, and 9. Sum the results of the products of (X - S) and (λT) or [(.2707*1) + (.1804*2) + (.0902*3) + (.0361*4) + (.0120*5) +

(.0034*6) + (.0009*7) + (.0002*8)] and the result is the long range expected number of backorders, or 1.1348.

Operational Rate (OR). This performance measure determines the probability that an aircraft lacks an essential recoverable component and is the product of the ready rates for all recoverable components for the aircraft (9:12-13). The expression is:

$$OR = \prod_{i=1}^{N} \sum_{X=0}^{S_i} p(X \mid \lambda_i T_i) \qquad (B.6)$$

This expression represents a no cannabilization policy. If we consider a full cannabilization policy for a certain number of aircraft (M), and quantity per aircraft (QPA), then the expression is:

$$OR = \prod_{i=1}^{N} \sum_{X=0}^{S_i + (QPA_i * M)} p(X \mid \lambda_i T_i) \qquad (B.7)$$

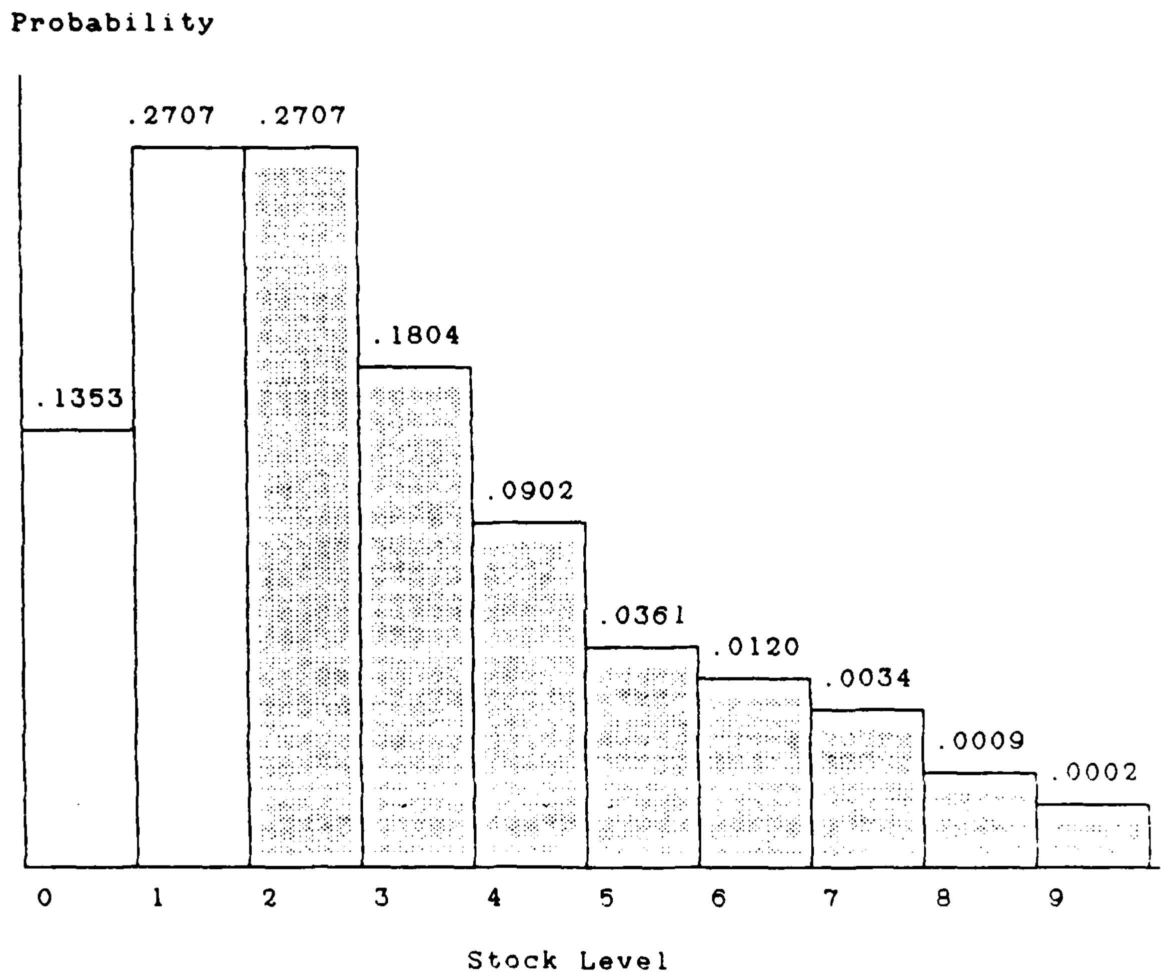


Figure 11. Poisson Distribution for λT = 2

Appendix C: Dyna-METRIC Computer Algorithm Outline with Formulas. Extracted from (26:10.1-17).

Introduction

Our discussion of Dyna-METRIC logic and computational processes will follow the basic programming sequence of the model in order for you to better understand how the model works.

ASSESSMENT MODE

TOP is the main routine of Dyna-METRIC:

- its function is to sequence the execution of subroutines to accomplish what the user has requested (by specifying options)

- assigns values to variables that establish the size of the data structure, i.e., array sizes and loop limits for:

  last day of war
  total number of bases
  total number of parts
  total number of LRUs
  total number of SRUs
  maximum SRUs per LRU
  maximum aircraft at any base

Read the Input Data

- SubroutineRDTOP reads in:
  title
  theater structure data
  base data
  CIRF data

- Subroutine RDSCEN reads in:
  aircraft
  sorties
  flying hours
  attrition
  missions by base
  maximum turn rate

-- Aircraft attrition is computed as this data is read in:

$$\text{Attr Day } N = \left(\text{Attr Day } N-1\right) + \left(\text{\# non-attr acft Day } N\right) \times \left(\text{sortie rate}\right) \times \left(\text{attr factor}\right)$$

-------------------------EXAMPLE-------------------------

Day N	Attr N-1	+	AC	*	Sor	*	Attr	=	Attr Day N	Non-Attr Aircraft
1	.00		48		3		.005		.72	--
2	.72		47		3		.005		1.43	47
3	1.43		47		3		.005		2.13	47
4	2.13		45		3		.005		2.81	45
5	2.81		45		3		.005		3.48	45
6	3.48		45		2.5		.005		4.04	44

-----------------------END EXAMPLE-----------------------

-- Compute cumulative aircraft and sorties each CIRF supports

- Subroutine RDPRT reads in:
  - LRU data
  - SRU data
  - Application fraction data

- Subroutine RDTST read in:
  - Indenture data
  - Test stand data

<u>Main Program Loop</u>
Executed once for each day of analysis:

- Subroutine RDSTK
  (reads stock levels for current time of analysis, if performance is based on input stock)

  Return to the main program

- Subroutine STKCRF
  (performs stockage and pipeline calculations at each CIRF)

  For each non-test stand LRU:

Compute mean pipelines, then backorders, and allocate backorders to the bases (in proportion to each bases' demands placed on the CIRF)

Buy stock to cover CIRF's pipeline, if option 2 is specified (no optimization, but a safety level is applied)

Compute SRU pipelines

Return to the main program

- Subroutine STKBS1 (for each base)

-- Subroutine SRUBAS (for each SRU)

For each day: calculate demands for parent LRUs, determine when LRU service is available and compute # of days until SRUs will arrive to be tested

Determine # of SRU demands

Determine when SRUs complete repairs

Determine volume of SRU peacetime pipeline which has not yet emptied

Calculate SRU pipeline for this day of analysis

Calculate data for LRUs awaiting SRUs (AWP) computations

Return to STKBS1

-- Subroutine LMBBAS
(computes pipeline quantities for non-test stand LRUs)

Calculate peacetime demands and initialize the peacetime pipelines (admin, in repair, off base and AWP)

Compute wartime *in-repair pipeline (fixed or random)*

Remember that the number of items in the pipeline is a function of λ (the demands/day) and Υ (the repair time)

<u>Demand Function</u>

In Dyna-METRIC, the demand function for an item, d(s), is a function of time (s)where:

$$d(s) = \left(\frac{\text{Failures}}{\text{Flying Hr}}\right) \times \left(\frac{\text{Flying Hrs @ t}}{\text{Sortie}}\right) \times \left(\text{QPA}\right) \times \left(\frac{\text{Sorties @ t}}{\text{Acft}}\right) \times \left(\text{Acft @ t}\right) \times \left(\begin{matrix}\text{Appl}\\ \text{Fraction}\end{matrix}\right)$$

<u>Repair Function</u>

The repair function for an item, F(s,t), is also a function of time: (t) = the current time and (s) = the time at which repair started.

F(t,s) = Prob [component entering repair at time (s) is still in repair at time (t)]

= Prob [Repair time > t-s when started at s]

FOR FIXED (DETERMINISTIC) REPAIR:

$$F(t,s) = \begin{cases} 1 & \text{if } t-s \text{ is } < RCT \\ 0 & \text{if } t-s \text{ is } \geq RCT \end{cases}$$

FOR RANDOM (EXPONENTIAL) REPAIR:

F(t,s) = 1 if t < Repair availability (Ra)

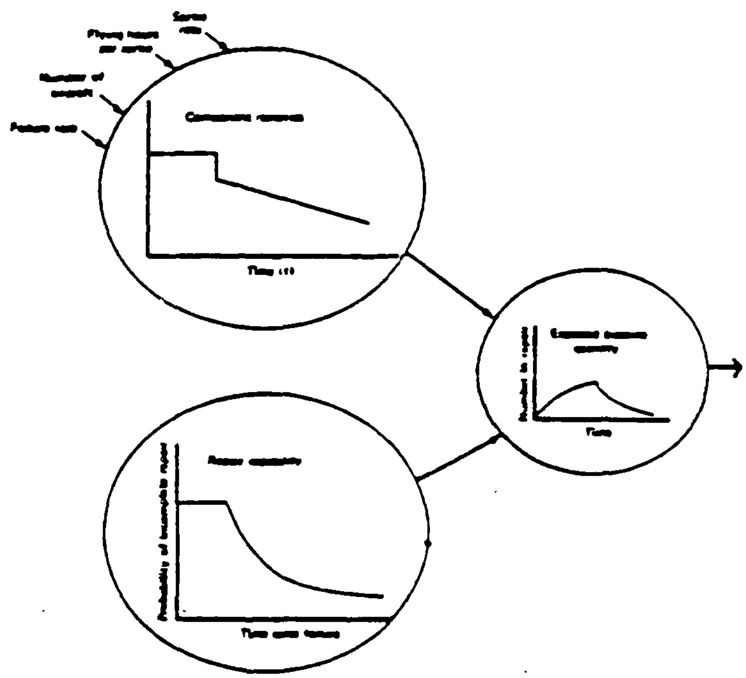
- - - - - - - - - - - - - - - - - - -

$$e^{-\left(\frac{t-RA}{RCT}\right)} \qquad \text{if } s < Ra \leq t$$

$$e^{-\left(\frac{t-s}{RCT}\right)} \quad \text{if } Ra \leq s < t$$

where RCT becomes the mean of the exponential distribution

<u>Demand function x Repair function = Pipeline:</u>



The expected number in the pipeline U(t) during a small interval from (s) to (t) is

$$U(t) = \sum_{s=0}^{t} d(s) \times F(t,s)$$

Expected number in the pipeline on day t = those demands that are in repair now at time (t)

The on-base and off-base segments of the pipelines are computed as follows:

Local Pipeline: $U1(t) = \sum_{s=0}^{t} PBR\ d(s)\ F_b(t,s)$

Remote Pipeline: $U2(t) = \sum_{s=0}^{t} NRTS\ d(s)\ F_c(t,s)$

The resulting Poisson pipeline distribution can be used to compute the probability of various pipeline sizes:

Prob [B or less components of type(i) in repair at time(t)]

$$= \sum_{b=0}^{B} \frac{U(t)^{B} e^{-U(t)}}{B!}$$ **Which represents a Nonhomogeneous Poisson Process**

This distribution is also used for establishing the confidence of achieving the desired level of performance.

Return to Stkbs1

Return to the main program

- Subroutine TEQBAS
  (calculates pipeline quantities for LRUs served by test stands located at non-CIRF bases)

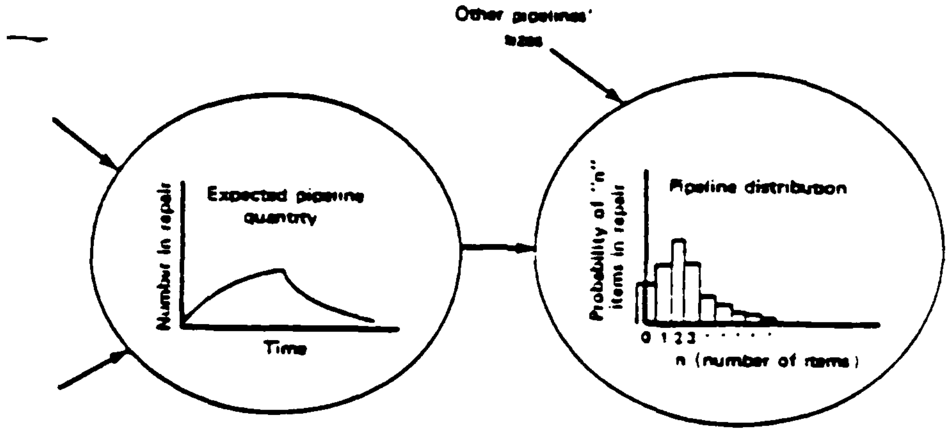
  Return to the main program

- Subroutine PERF
  (computes performance based on the computed pipeline quantities and the given stock levels)

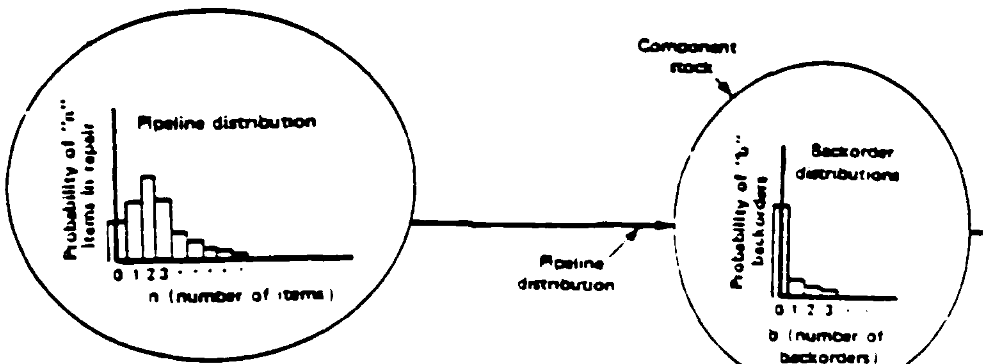
  Compute the integer target number of NMCS AC

  Loop through the LRUs looking for those affecting performance (i.e., check mission essentiality)

    Gettrm: computes the pipeline distributions for items with a given mean pipeline size and Variance/Mean ratio



    Bopmf: maps the pipeline distribution into a backorder distribution by shifting the pipeline distribution leftward by the number of items carried in stock



    Cdfsta: computes the mean and standard deviation of the given cumulative distribution

Compute no cann and full cann NMCS

EXPECTED NO-CANN NMCS, where QPA = 1 for all components (i):

E(NMCS) = # Aircraft (Prob [not missing any parts])

$$E(NMCS) = NA \left(1 - \prod_{i=1}^{N} \left(\frac{(1 - E(Bi))}{NA} \right) \right)$$

Total number of aircraft times the probability an aircraft is not missing any of (i) parts.

EXPECTED NO-CANN NMCS, where QPA > 1 for some of (i) components:

E(NMCS) = # Aircraft (Prob [an aircraft is missing some application of one of its parts])

$$= NA \left(1 - \prod_{i=1}^{N} \sum_{y=0}^{NA*Q} \text{Prob[aircraft have no shortages of i when y shortages exist]} \right)$$

$$= NA \left(1 - \prod_{i=1}^{N} \sum_{y=0}^{NA*Q} \frac{\left(\text{\# combinations of Q from good installed parts}\right)}{\left(\text{\# combinations of Q from all installed parts}\right)} P(Bi=y) \right)$$

$$= NA \left(1 - \prod_{i=1}^{N} \sum_{y=0}^{NA*Q} \frac{\binom{Qi*NA-y}{Qi}}{\binom{Qi*NA}{Qi}} P(Bi=y) \right)$$

where $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

The model computes all the possible combinations of having a backorder for a single application of each part, given there are already y backorders for that part. This computation is used to derive the probability an aircraft will be missing at least on application of one of its components. This probability multiplied by the fleet size (NA) results in the expected number of NMCS aircraft.

EXPECTED FULL-CANN NMCS, where QPA > 1 for some of (i) components:

$$E(NMCS) = \sum_{j=0}^{NA-1} \left(1 - \text{Prob}\,[NMCS \leq j^*Q]\right)$$

$$= \sum_{j=0}^{NA-1} \left(1 - \prod_{i=1}^{N} \left(\text{Prob}\,[\text{backorders} < j^*Q]\right)\right)$$

$$= \sum_{j=0}^{NA-1} \left(1 - \prod_{i=1}^{N} \left(\text{Prob}\,[\text{demands} < Si + j^*Q]\right)\right)$$

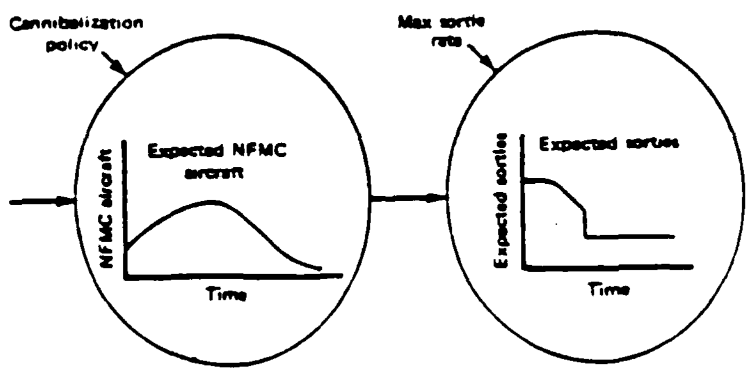
$$= \sum_{j=0}^{NA-1} \left(1 - \prod_{i=1}^{N} \left(\sum_{k=0}^{Si+jQ} \frac{Ui^{k}\, e^{-Ui}}{k!}\right)\right)$$

where Ui = number of LRUs in the pipeline and k = stock level

* ***KEY IDEA:***

The expected number of NMC acft is a function of the expected number of backorders at time (t)

Compute sortie statistics



Given a maximum sortie rate, MSR(t), then

$$E\ (\text{FMC sorties @ t}) = \sum_{X=1}^{NA} \left(X * SR * \text{Prob}(X\ \text{FMC})\right)$$

$$\text{where } SR = MSR(t) \text{ when } X < \frac{\text{planned sorties}}{\text{max rate}}$$

$$\text{else} = \frac{\text{planned sorties}}{X}$$

This results in the minimum of 1) the planned sorties and 2) the most sorties that can be flown with available FMC aircraft.

-------------------------EXAMPLE-------------------------

Given: max sortie rate = 3, planned sortie rate = 2
NA (# aircraft) = 4, planned sorties = 8
Prob(X FMC) a computed distribution =

X	Prob
0	.1
1	.3
2	.3
3	.2
4	.1

$$E(\text{sorties}) = \sum_{x=1}^{NA} \left(X * SR * \text{Prob}(X\ \text{FMC})\right)$$

	X		SR		P(X FMC)		
=	1	*	3	*	.3	=	.9
+	2	*	3	*	.3	=	1.8
+	3	*	8/3	*	.2	=	1.6
+	4	*	8/4	*	.1	=	.8
							5.1

--------------- -------END EXAMPLE-------------------------

* **KEY IDEA:**

- **FMC sorties is, in part, a function of MSR**

- **MSR(t) can be a "questionable" input**

- **E(# NMC AC) may be a better measure of performance than expected sorties**

Return to the main program

- Subroutine PROBLM:
  (identifies those LRUs whose stockage and repair processes most dramatically affect overall combat capability)

  The model will generate a "problem parts" list for day (t)

  - List of LRUs with the highest probability of restricting ability to accomplish plan (assuming full-cann)

  - Length of list dependent upon user-specified confidence level (arbitrary)

  - Makeup of the list may vary over time as variables change

  Return to the main program

REQUIREMENTS MODE

- Subroutine STKTEQ
  (computes the test equipment shortfall, if any, and buys test stands to cover the shortage)

  Return to the main program

- Subroutine STKBS2
  (completes the computations for stockage and performance at the standard (nonCIRF) bases)

  Loop through the bases, buying additional stock according to the options selected

  Read the pipeline data stored by Stkbas1

  If option 3 is specified, calculate target number of NMCS AC, and buy stock to achieve that target

If option 4 is specified, perform a marginal analysis to optimally stock base LRUs for the desired level of performance

Buy LRU with best increase and loop to next part

MINIMIZE: $\sum_{j=1}^{M} \sum_{i=1}^{N} Ci\ Sij$

Minimize the cost of the stock for all items (i through N) at all bases (j through M)

SUBJECT TO:

Prob [# NMC acft $\leq$ Y] $\geq$ $\left(\begin{matrix}\text{confidence level}\\ \text{specified}\end{matrix}\right)$

Meeting the requested NMC goal with the requested level of confidence

Where Prob [NMC acft $\leq$ Y]

$= \prod_{i=1}^{N}$ Prob [less than (Y)Qi backorders for item (i) at time (t)]

$= \prod_{i=1}^{N} \sum_{Y=0}^{Si+QiY}$ Prob [exactly Y failures of item(i) at time(t)]

NOTE: In the requirements mode, the ratio of support among parts may differ over time

If performance is to be computed based on stock purchased, compute performance (see Subroutine PERF)

Return to the main program

- Subroutine STKSRU
  (performs SRU stockage calculations for bases and CIRFs for each day of analysis)

If option 6 is specified, calculate target number of NMCS AC, and buy stock to achieve that target

If option 7 is specified, perform a marginal analysis to optimally stock for the desired level of performance

Buy SRU with best AWP improvement and loop to next part

Return to the main program

- Subroutine CSTUPD
  (updates the running stock cost totals)

  Return to the main program

- Subroutine OUTP
  (outputs a table of cumulative stockage costs)

  Return to the main program loop (for another day of analysis)

END OF PROGRAM

DYNA-METRIC ASSUMPTIONS AND LIMITATIONS

Repair Assumptions

- "Ample Repair" assumes there are sufficient resources to repair components within the specified RCT.

  -- When close to maximum maintenance capacity, the model overstates capability because Awaiting Maintenance delays are not addressed

  -- Degree of overstatement is function of the size of the pipeline

- Stationary repair process

  -- The demand process is independent of the repair process

  -- Repair surges and slowdowns cannot be evaluated

- Repair process for each part is identical at all bases and CIRFs

  -- The single RCT specified for each part represents the repair process for that part

  -- Limited workaround involves using CIRF and/or base administrative delay times to differentiate between base and CIRF processes

Demand Generation Assumptions

- Component failures are a function of flying hour intensity

  -- Consumption of some items (such as tires, gun items and test equipment) are not driven by flying hours

  -- An approximate ("equivalent") flying hour demand rate must be derived

- Component failures are generated by requested flying hours rather than those expected to be flown by the computed number of FMC aircraft

  -- Assumes some PMC aircraft are used to fly the requested sorties

  -- In actuality, if very few PMC aircraft are available, then demands are overestimated and capability is underestimated

  -- If expected sorties vary from the requested by approximately 20%, the results are questionable

  -- To get a reasonable measure of capability, you must manually feedback expected FMC sorties as the demanded sorties

- Demands are not affected by environmental, organizational, and other differences between bases

  -- Demand rates per LRU are the same for all bases

  -- This may be partially addressed by unit level assessments

- When demands are assumed Poisson (which is usually the case)

  -- Equivalent to saying LRU lifetimes are exponential, that is, there is no wearout

  -- Components in burn-in or wearout will fail more often than fleetwide long-term data might indicate, and model overestimates capability for these parts

<u>Support Assumptions</u>

- No lateral resupply

  -- Lateral resupply is difficult to integrate into model

  -- Underestimates capability if lateral supply actually exits

- Cannibalization policy is "All or Nothing"

  - No-cann understates capability

  - Full-cann overstates capability

- Cannibalizations occur instantly

  -- Additional repair time for cannibalizations is not accounted for, thereby overstating capability

- Max sortie rate cannot vary between bases

  - Turn rate differentials from base to base are not addressed

Other Assumptions

- Only semi-homogeneous aircraft can be modeled at each base

  -- Assumed cannibalization process prevents direct analysis of collocated multiple MDS with some shared common stock

  -- Basically one MD per base .... If some fraction of aircraft have a set of additive LRU'S, then use "% application" feature

  -- Workaround splits single base into several "bases"..... one for each unique MDS

- All aircraft are fully mission capable at start of scenario

- Expected FMC Sorties are unconstrained except by the expected number of FMC aircraft

  -- The demanded sorties will be satisfied subject only to maximum sortie rate limitations

  -- Operational and flight line constraints are ignored

- Only two levels of indenture

  -- If third level of indenture exists (i.e., subSRU or Bit/Piece) and inventory is not available, then SRU repair process will be overstated and capability is overstated (unless AWP time is included in RCT)

- No condemnation of LRUs

  -- Battle damage or failures are always repaired somewhere

- Model addresses only LRUs as problem parts; there is no performance data on indentured SRUs

Appendix D: Normal, Poisson, and Partial Expectation Tables. Extracted from (6) and (38).

NORMAL DISTRIBUTION AND RELATED FUNCTIONS

x	$F(x)$	$1 - F(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
.00	.5000	.5000	.3989	−.0000	−.3989	.0000	1.1968
.01	.5040	.4960	.3989	−.0040	−.3989	.0120	1.1965
.02	.5080	.4920	.3989	−.0080	−.3987	.0239	1.1956
.03	.5120	.4880	.3988	−.0120	−.3984	.0359	1.1941
.04	.5160	.4840	.3986	−.0159	−.3980	.0478	1.1920
.05	.5199	.4801	.3984	−.0199	−.3975	.0597	1.1894
.06	.5239	.4761	.3982	−.0239	−.3968	.0716	1.1861
.07	.5279	.4721	.3980	−.0279	−.3960	.0834	1.1822
.08	.5319	.4681	.3977	−.0318	−.3951	.0952	1.1778
.09	.5359	.4641	.3973	−.0358	−.3941	.1070	1.1727
.10	.5398	.4602	.3970	−.0397	−.3930	.1187	1.1671
.11	.5438	.4562	.3965	−.0436	−.3917	.1303	1.1609
.12	.5478	.4522	.3961	−.0475	−.3904	.1419	1.1541
.13	.5517	.4483	.3956	−.0514	−.3889	.1534	1.1468
.14	.5557	.4443	.3951	−.0553	−.3873	.1648	1.1389
.15	.5596	.4404	.3945	−.0592	−.3856	.1762	1.1304
.16	.5636	.4364	.3939	−.0630	−.3838	.1874	1.1214
.17	.5675	.4325	.3932	−.0668	−.3819	.1986	1.1118
.18	.5714	.4286	.3925	−.0707	−.3798	.2097	1.1017
.19	.5753	.4247	.3918	−.0744	−.3777	.2206	1.0911
.20	.5793	.4207	.3910	−.0782	−.3754	.2315	1.0799
.21	.5832	.4168	.3902	−.0820	−.3730	.2422	1.0682
.22	.5871	.4129	.3894	−.0857	−.3706	.2529	1.0560
.23	.5910	.4090	.3885	−.0894	−.3680	.2634	1.0434
.24	.5948	.4052	.3876	−.0930	−.3653	.2737	1.0302
.25	.5987	.4013	.3867	−.0967	−.3625	.2840	1.0165
.26	.6026	.3974	.3857	−.1003	−.3596	.2941	1.0024
.27	.6064	.3936	.3847	−.1039	−.3566	.3040	0.9878
.28	.6103	.3897	.3836	−.1074	−.3535	.3138	0.9727
.29	.6141	.3859	.3825	−.1109	−.3504	.3235	0.9572
.30	.6179	.3821	.3814	−.1144	−.3471	.3330	0.9413
.31	.6217	.3783	.3802	−.1179	−.3437	.3423	0.9250
.32	.6255	.3745	.3790	−.1213	−.3402	.3515	0.9082
.33	.6293	.3707	.3778	−.1247	−.3367	.3605	0.8910
.34	.6331	.3669	.3765	−.1280	−.3330	.3693	0.8735
.35	.6368	.3632	.3752	−.1313	−.3293	.3779	0.8556
.36	.6406	.3594	.3739	−.1346	−.3255	.3864	0.8373
.37	.6443	.3557	.3725	−.1378	−.3216	.3947	0.8186
.38	.6480	.3520	.3712	−.1410	−.3176	.4028	0.7996
.39	.6517	.3483	.3697	−.1442	−.3135	.4107	0.7803
.40	.6554	.3446	.3683	−.1473	−.3094	.4184	0.7607
.41	.6591	.3409	.3668	−.1504	−.3051	.4259	0.7408
.42	.6628	.3372	.3653	−.1534	−.3008	.4332	0.7206
.43	.6664	.3336	.3637	−.1564	−.2965	.4403	0.7001
.44	.6700	.3300	.3621	−.1593	−.2920	.4472	0.6793
.45	.6736	.3264	.3605	−.1622	−.2875	.4539	0.6583
.46	.6772	.3228	.3589	−.1651	−.2830	.4603	0.6371
.47	.6808	.3192	.3572	−.1679	−.2783	.4666	0.6156
.48	.6844	.3156	.3555	−.1707	−.2736	.4727	0.5940
.49	.6879	.3121	.3538	−.1734	−.2689	.4785	0.5721
.50	.6915	.3085	.3521	−.1760	−.2641	.4841	0.5501

NORMAL DISTRIBUTION AND RELATED FUNCTIONS

x	$F(x)$	$1 - F(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
.50	.6915	.3085	.3521	−.1760	−.2641	.4841	.5501
.51	.6950	.3050	.3503	−.1787	−.2592	.4895	.5279
.52	.6985	.3015	.3485	−.1812	−.2543	.4947	.5056
.53	.7019	.2981	.3467	−.1837	−.2493	.4996	.4831
.54	.7054	.2946	.3448	−.1862	−.2443	.5043	.4605
.55	.7088	.2912	.3429	−.1886	−.2392	.5088	.4378
.56	.7123	.2877	.3410	−.1920	−.2341	.5131	.4150
.57	.7157	.2843	.3391	−.1933	−.2289	.5171	.3921
.58	.7190	.2810	.3372	−.1956	−.2238	.5209	.3691
.59	.7224	.2776	.3352	−.1978	−.2185	.5245	.3461
.60	.7257	.2743	.3332	−.1999	−.2133	.5278	.3231
.61	.7291	.2709	.3312	−.2020	−.2080	.5309	.3000
.62	.7324	.2676	.3292	−.2041	−.2027	.5338	.2770
.63	.7357	.2643	.3271	−.2061	−.1973	.5365	.2539
.64	.7389	.2611	.3251	−.2080	−.1919	.5389	.2309
.65	.7422	.2578	.3230	−.2099	−.1865	.5411	.2078
.66	.7454	.2546	.3209	−.2118	−.1811	.5431	.1849
.67	.7486	.2514	.3187	−.2136	−.1757	.5448	.1620
.68	.7517	.2483	.3166	−.2153	−.1702	.5463	.1391
.69	.7549	.2451	.3144	−.2170	−.1647	.5476	.1164
.70	.7580	.2420	.3123	−.2186	−.1593	.5486	.0937
.71	.7611	.2389	.3101	−.2201	−.1538	.5495	.0712
.72	.7642	.2358	.3079	−.2217	−.1483	.5501	.0487
.73	.7673	.2327	.3056	−.2231	−.1428	.5504	.0265
.74	.7704	.2296	.3034	−.2245	−.1373	.5506	.0043
.75	.7734	.2266	.3011	−.2259	−.1318	.5505	−.0176
.76	.7764	.2236	.2989	−.2271	−.1262	.5502	−.0394
.77	.7794	.2206	.2966	−.2284	−.1207	.5497	−.0611
.78	.7823	.2177	.2943	−.2296	−.1153	.5490	−.0825
.79	.7852	.2148	.2920	−.2307	−.1098	.5481	−.1037
.80	.7881	.2119	.2897	−.2318	−.1043	.5469	−.1247
.81	.7910	.2090	.2874	−.2328	−.0988	.5456	−.1455
.82	.7939	.2061	.2850	−.2337	−.0934	.5440	−.1660
.83	.7967	.2033	.2827	−.2346	−.0880	.5423	−.1862
.84	.7995	.2005	.2803	−.2355	−.0825	.5403	−.2063
.85	.8023	.1977	.2780	−.2363	−.0771	.5381	−.2260
.86	.8051	.1949	.2756	−.2370	−.0718	.5358	−.2455
.87	.8078	.1922	.2732	−.2377	−.0664	.5332	−.2646
.88	.8106	.1894	.2709	−.2384	−.0611	.5305	−.2835
.89	.8133	.1867	.2685	−.2389	−.0558	.5276	−.3021
.90	.8159	.1841	.2661	−.2395	−.0506	.5245	−.3203
.91	.8186	.1814	.2637	−.2400	−.0453	.5212	−.3383
.92	.8212	.1788	.2613	−.2404	−.0401	.5177	−.3559
.93	.8238	.1762	.2589	−.2408	−.0350	.5140	−.3731
.94	.8264	.1736	.2565	−.2411	−.0299	.5102	−.3901
.95	.8289	.1711	.2541	−.2414	−.0248	.5062	−.4066
.96	.8315	.1685	.2516	−.2416	−.0197	.5021	−.4228
.97	.8340	.1660	.2492	−.2417	−.0147	.4978	−.4387
.98	.8365	.1635	.2468	−.2419	−.0098	.4933	−.4541
.99	.8389	.1611	.2444	−.2420	−.0049	.4887	−.4692
1.00	.8413	.1587	.2420	−.2420	.0000	.4839	−.4839

NORMAL DISTRIBUTION AND RELATED FUNCTIONS

x	$F(x)$	$1 - F(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
1.00	.8413	.1587	.2420	−.2420	.0000	.4839	−.4839
1.01	.8438	.1562	.2396	−.2420	.0048	.4790	−.4983
1.02	.8461	.1539	.2371	−.2419	.0096	.4740	−.5122
1.03	.8485	.1515	.2347	−.2418	.0143	.4688	−.5257
1.04	.8508	.1492	.2323	−.2416	.0190	.4635	−.5389
1.05	.8531	.1469	.2299	−.2414	.0236	.4580	−.5516
1.06	.8554	.1446	.2275	−.2411	.0281	.4524	−.5639
1.07	.8577	.1423	.2251	−.2408	.0326	.4467	−.5758
1.08	.8599	.1401	.2227	−.2405	.0371	.4409	−.5873
1.09	.8621	.1379	.2203	−.2401	.0414	.4350	−.5984
1.10	.8643	.1357	.2179	−.2396	.0458	.4290	−.6091
1.11	.8665	.1335	.2155	−.2392	.0500	.4228	−.6193
1.12	.8686	.1314	.2131	−.2386	.0542	.4166	−.6292
1.13	.8708	.1292	.2107	−.2381	.0583	.4102	−.6386
1.14	.8729	.1271	.2083	−.2375	.0624	.4038	−.6476
1.15	.8749	.1251	.2059	−.2368	.0664	.3973	−.6561
1.16	.8770	.1230	.2036	−.2361	.0704	.3907	−.6643
1.17	.8790	.1210	.2012	−.2354	.0742	.3840	−.6720
1.18	.8810	.1190	.1989	−.2347	.0780	.3772	−.6792
1.19	.8830	.1170	.1965	−.2339	.0818	.3704	−.6861
1.20	.8849	.1151	.1942	−.2330	.0854	.3635	−.6926
1.21	.8869	.1131	.1919	−.2322	.0890	.3566	−.6986
1.22	.8888	.1112	.1895	−.2312	.0926	.3496	−.7042
1.23	.8907	.1093	.1872	−.2303	.0960	.3425	−.7094
1.24	.8925	.1075	.1849	−.2293	.0994	.3354	−.7141
1.25	.8944	.1056	.1826	−.2283	.1027	.3282	−.7185
1.26	.8962	.1038	.1804	−.2273	.1060	.3210	−.7224
1.27	.8980	.1020	.1781	−.2262	.1092	.3138	−.7259
1.28	.8997	.1003	.1758	−.2251	.1123	.3065	−.7291
1.29	.9015	.0985	.1736	−.2240	.1153	.2992	−.7318
1.30	.9032	.0968	.1714	−.2228	.1182	.2918	−.7341
1.31	.9049	.0951	.1691	−.2216	.1211	.2845	−.7361
1.32	.9066	.0934	.1669	−.2204	.1239	.2771	−.7376
1.33	.9082	.0918	.1647	−.2191	.1267	.2697	−.7388
1.34	.9099	.0901	.1626	−.2178	.1293	.2624	−.7395
1.35	.9115	.0885	.1604	−.2165	.1319	.2550	−.7399
1.36	.9131	.0869	.1582	−.2152	.1344	.2476	−.7400
1.37	.9147	.0853	.1561	−.2138	.1369	.2402	−.7396
1.38	.9162	.0838	.1539	−.2125	.1392	.2328	−.7389
1.39	.9177	.0823	.1518	−.2110	.1415	.2254	−.7378
1.40	.9192	.0808	.1497	−.2096	.1437	.2180	−.7364
1.41	.9207	.0793	.1476	−.2082	.1459	.2107	−.7347
1.42	.9222	.0778	.1456	−.2067	.1480	.2033	−.7326
1.43	.9236	.0764	.1435	−.2052	.1500	.1960	−.7301
1.44	.9251	.0749	.1415	−.2037	.1519	.1887	−.7274
1.45	.9265	.0735	.1394	−.2022	.1537	.1815	−.7243
1.46	.9279	.0721	.1374	−.2006	.1555	.1742	−.7209
1.47	.9292	.0708	.1354	−.1991	.1572	.1670	−.7172
1.48	.9306	.0694	.1334	−.1975	.1588	.1599	−.7132
1.49	.9319	.0681	.1315	−.1959	.1604	.1528	−.7089
1.50	.9332	.0668	.1295	−.1943	.1619	.1457	−.7043

NORMAL DISTRIBUTION AND RELATED FUNCTIONS

x	$F(x)$	$1 - F(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
1.50	.9332	.0668	.1295	−.1943	.1619	.1457	−.7043
1.51	.9345	.0655	.1276	−.1927	.1633	.1387	−.6994
1.52	.9357	.0643	.1257	−.1910	.1647	.1317	−.6942
1.53	.9370	.0630	.1238	−.1894	.1660	.1248	−.6888
1.54	.9382	.0618	.1219	−.1877	.1672	.1180	−.6831
1.55	.9394	.0606	.1200	−.1860	.1683	.1111	−.6772
1.56	.9406	.0594	.1182	−.1843	.1694	.1044	−.6710
1.57	.9418	.0582	.1163	−.1826	.1704	.0977	−.6646
1.58	.9429	.0571	.1145	−.1809	.1714	.0911	−.6580
1.59	.9441	.0559	.1127	−.1792	.1722	.0846	−.6511
1.60	.9452	.0548	.1109	−.1775	.1730	.0781	−.6441
1.61	.9463	.0537	.1092	−.1757	.1738	.0717	−.6368
1.62	.9474	.0526	.1074	−.1740	.1745	.0654	−.6293
1.63	.9484	.0516	.1057	−.1723	.1751	.0591	−.6216
1.64	.9495	.0505	.1040	−.1705	.1757	.0529	−.6138
1.65	.9505	.0495	.1023	−.1687	.1762	.0468	−.6057
1.66	.9515	.0485	.1006	−.1670	.1766	.0408	−.5975
1.67	.9525	.0475	.0989	−.1652	.1770	.0349	−.5891
1.68	.9535	.0465	.0973	−.1634	.1773	.0290	−.5806
1.69	.9545	.0455	.0957	−.1617	.1776	.0233	−.5720
1.70	.9554	.0446	.0940	−.1599	.1778	.0176	−.5632
1.71	.9564	.0436	.0925	−.1581	.1779	.0120	−.5542
1.72	.9573	.0427	.0909	−.1563	.1780	.0065	−.5452
1.73	.9582	.0418	.0893	−.1546	.1780	.0011	−.5360
1.74	.9591	.0409	.0878	−.1528	.1780	−.0042	−.5267
1.75	.9599	.0401	.0863	−.1510	.1780	−.0094	−.5173
1.76	.9608	.0392	.0848	−.1492	.1778	−.0146	−.5079
1.77	.9616	.0384	.0833	−.1474	.1777	−.0196	−.4983
1.78	.9625	.0375	.0818	−.1457	.1774	−.0245	−.4887
1.79	.9633	.0367	.0804	−.1439	.1772	−.0294	−.4789
1.80	.9641	.0359	.0790	−.1421	.1769	−.0341	−.4692
1.81	.9649	.0351	.0775	−.1403	.1765	−.0388	−.4593
1.82	.9656	.0344	.0761	−.1386	.1761	−.0433	−.4494
1.83	.9664	.0336	.0748	−.1368	.1756	−.0477	−.4395
1.84	.9671	.0329	.0734	−.1351	.1751	−.0521	−.4295
1.85	.9678	.0322	.0721	−.1333	.1746	−.0563	−.4195
1.86	.9686	.0314	.0707	−.1316	.1740	−.0605	−.4095
1.87	.9693	.0307	.0694	−.1298	.1734	−.0645	−.3995
1.88	.9699	.0301	.0681	−.1281	.1727	−.0685	−.3894
1.89	.9706	.0294	.0669	−.1264	.1720	−.0723	−.3793
1.90	.9713	.0287	.0656	−.1247	.1713	−.0761	−.3693
1.91	.9719	.0281	.0344	−.1230	.1705	−.0797	−.3592
1.92	.9726	.0274	.0632	−.1213	.1697	−.0832	−.3492
1.93	.9732	.0268	.0620	−.1196	.1688	−.0867	−.3392
1.94	.9738	.0262	.0608	−.1179	.1679	−.0900	−.3292
1.95	.9744	.0256	.0596	−.1162	.1670	−.0933	−.3192
1.96	.9750	.0250	.0584	−.1145	.1661	−.0964	−.3093
1.97	.9756	.0244	.0573	−.1129	.1651	−.0994	−.2994
1.98	.9761	.0239	.0562	−.1112	.1641	−.1024	−.2895
1.99	.9767	.0233	.0551	−.1096	.1630	−.1052	−.2797
2.00	.9772	.0228	.0540	−.1080	.1620	−.1080	−.2700

x	$F(x)$	$1 - F(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
2.00	.9773	.0227	.0540	−.1080	.1620	−.1080	−.2700
2.01	.9778	.0222	.0529	−.1064	.1609	−.1106	−.2603
2.02	.9783	.0217	.0519	−.1048	.1598	−.1132	−.2506
2.03	.9788	.0212	.0508	−.1032	.1586	−.1157	−.2411
2.04	.9793	.0207	.0498	−.1016	.1575	−.1180	−.2316
2.05	.9798	.0202	.0488	−.1000	.1563	−.1203	−.2222
2.06	.9803	.0197	.0478	−.0985	.1550	−.1225	−.2129
2.07	.9808	.0192	.0468	−.0969	.1538	−.1245	−.2036
2.08	.9812	.0188	.0459	−.0954	.1526	−.1265	−.1945
2.09	.9817	.0183	.0449	−.0939	.1513	−.1284	−.1854
2.10	.9821	.0179	.0440	−.0924	.1500	−.1302	−.1765
2.11	.9826	.0174	.0431	−.0909	.1487	−.1320	−.1676
2.12	.9830	.0170	.0422	−.0894	.1474	−.1336	−.1588
2.13	.9834	.0166	.0413	−.0879	.1460	−.1351	−.1502
2.14	.9838	.0162	.0404	−.0865	.1446	−.1366	−.1416
2.15	.9842	.0158	.0396	−.0850	.1433	−.1380	−.1332
2.16	.9846	.0154	.0387	−.0836	.1419	−.1393	−.1249
2.17	.9850	.0150	.0379	−.0822	.1405	−.1405	−.1167
2.18	.9854	.0146	.0371	−.0808	.1391	−.1416	−.1086
2.19	.9857	.0143	.0363	−.0794	.1377	−.1426	−.1006
2.20	.9861	.0139	.0355	−.0780	.1362	−.1436	−.0927
2.21	.9864	.0136	.0347	−.0767	.1348	−.1445	−.0850
2.22	.9868	.0132	.0339	−.0754	.1333	−.1453	−.0774
2.23	.9871	.0129	.0332	−.0740	.1319	−.1460	−.0700
2.24	.9875	.0125	.0325	−.0727	.1304	−.1467	−.0626
2.25	.9878	.0122	.0317	−.0714	.1289	−.1473	−.0554
2.26	.9881	.0119	.0310	−.0701	.1275	−.1478	−.0484
2.27	.9884	.0116	.0303	−.0689	.1260	−.1483	−.0414
2.28	.9887	.0113	.0297	−.0676	.1245	−.1486	−.0346
2.29	.9890	.0110	.0290	−.0664	.1230	−.1490	−.0279
2.30	.9893	.0107	.0283	−.0652	.1215	−.1492	−.0214
2.31	.9896	.0104	.0277	−.0639	.1200	−.1494	−.0150
2.32	.9898	.0102	.0270	−.0628	.1185	−.1495	−.0088
2.33	.9901	.0099	.0264	−.0616	.1170	−.1496	−.0027
2.34	.9904	.0096	.0258	−.0604	.1155	−.1496	.0033
2.35	.9906	.0094	.0252	−.0593	.1141	−.1495	.0092
2.36	.9909	.0091	.0246	−.0581	.1126	−.1494	.0149
2.37	.9911	.0089	.0241	−.0570	.1111	−.1492	.0204
2.38	.9913	.0087	.0235	−.0559	.1096	−.1490	.0258
2.39	.9916	.0084	.0229	−.0548	.1081	−.1487	.0311
2.40	.9918	.0082	.0224	−.0538	.1066	−.1483	.0362
2.41	.9920	.0080	.0219	−.0527	.1051	−.1480	.0412
2.42	.9922	.0078	.0213	−.0516	.1036	−.1475	.0461
2.43	.9925	.0075	.0208	−.0506	.1022	−.1470	.0508
2.44	.9927	.0073	.0203	−.0496	.1007	−.1465	.0554
2.45	.9929	.0071	.0198	−.0486	.0992	−.1459	.0598
2.46	.9931	.0069	.0194	−.0476	.0978	−.1453	.0641
2.47	.9932	.0068	.0189	−.0467	.0963	−.1446	.0683
2.48	.9934	.0066	.0184	−.0457	.0949	−.1439	.0723
2.49	.9936	.0064	.0180	−.0448	.0935	−.1432	.0762
2.50	.9938	.0062	.0175	−.0438	.0920	−.1424	.0800

NORMAL DISTRIBUTION AND RELATED FUNCTIONS

x	$F(x)$	$1-F(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
2.50	.9938	.0062	.0175	−.0438	.0920	−.1424	.0800
2.51	.9940	.0060	.0171	−.0429	.0906	−.1416	.0836
2.52	.9941	.0059	.0167	−.0420	.0892	−.1408	.0871
2.53	.9943	.0057	.0163	−.0411	.0878	−.1399	.0905
2.54	.9945	.0055	.0158	−.0403	.0864	−.1389	.0937
2.55	.9946	.0054	.0155	−.0394	.0850	−.1380	.0968
2.56	.9948	.0052	.0151	−.0386	.0836	−.1370	.0998
2.57	.9949	.0051	.0147	−.0377	.0823	−.1360	.1027
2.58	.9951	.0049	.0143	−.0369	.0809	−.1350	.1054
2.59	.9952	.0048	.0139	−.0361	.0796	−.1339	.1080
2.60	.9953	.0047	.0136	−.0353	.0782	−.1328	.1105
2.61	.9955	.0045	.0132	−.0345	.0769	−.1317	.1129
2.62	.9956	.0044	.0129	−.0338	.0756	−.1305	.1152
2.63	.9957	.0043	.0126	−.0330	.0743	−.1294	.1173
2.64	.9959	.0041	.0122	−.0323	.0730	−.1282	.1194
2.65	.9960	.0040	.0119	−.0316	.0717	−.1270	.1213
2.66	.9961	.0039	.0116	−.0309	.0705	−.1258	.1231
2.67	.9962	.0038	.0113	−.0302	.0692	−.1245	.1248
2.68	.9963	.0037	.0110	−.0295	.0680	−.1233	.1264
2.69	.9964	.0036	.0107	−.0288	.0668	−.1220	.1279
2.70	.9965	.0035	.0104	−.0281	.0656	−.1207	.1293
2.71	.9966	.0034	.0101	−.0275	.0644	−.1194	.1306
2.72	.9967	.0033	.0099	−.0269	.0632	−.1181	.1317
2.73	.9968	.0032	.0096	−.0262	.0620	−.1168	.1328
2.74	.9969	.0031	.0093	−.0256	.0608	−.1154	.1338
2.75	.9970	.0030	.0091	−.0250	.0597	−.1141	.1347
2.76	.9971	.0029	.0088	−.0244	.0585	−.1127	.1356
2.77	.9972	.0028	.0086	−.0238	.0574	−.1114	.1363
2.78	.9973	.0027	.0084	−.0233	.0563	−.1100	.1369
2.79	.9974	.0026	.0081	−.0227	.0552	−.1087	.1375
2.80	.9974	.0026	.0079	−.0222	.0541	−.1073	.1379
2.81	.9975	.0025	.0077	−.0216	.0531	−.1059	.1383
2.82	.9976	.0024	.0075	−.0211	.0520	−.1045	.1386
2.83	.9977	.0023	.0073	−.0206	.0510	−.1031	.1389
2.84	.9977	.0023	.0071	−.0201	.0500	−.1017	.1390
2.85	.9978	.0022	.0069	−.0196	.0490	−.1003	.1391
2.86	.9979	.0021	.0067	−.0191	.0480	−.0990	.1391
2.87	.9979	.0021	.0065	−.0186	.0470	−.0976	.1391
2.88	.9980	.0020	.0063	−.0182	.0460	−.0962	.1389
2.89	.9981	.0019	.0061	−.0177	.0451	−.0948	.1388
2.90	.9981	.0019	.0060	−.0173	.0441	−.0934	.1385
2.91	.9982	.0018	.0058	−.0168	.0432	−.0920	.1382
2.92	.9982	.0018	.0056	−.0164	.0423	−.0906	.1378
2.93	.9983	.0017	.0055	−.0160	.0414	−.0893	.1374
2.94	.9984	.0016	.0053	−.0156	.0405	−.0879	.1369
2.95	.9984	.0016	.0051	−.0152	.0396	−.0865	.1364
2.96	.9985	.0015	.0050	−.0148	.0388	−.0852	.1358
2.97	.9985	.0015	.0048	−.0144	.0379	−.0838	.1352
2.98	.9986	.0014	.0047	−.0140	.0371	−.0825	.1345
2.99	.9986	.0014	.0046	−.0137	.0363	−.0811	.1337
3.00	.9987	.0013	.0044	−.0133	.0355	−.0798	.1330

NORMAL DISTRIBUTION AND RELATED FUNCTIONS

z	$F(z)$	$1 - F(z)$	$f(z)$	$f'(z)$	$f''(z)$	$f'''(z)$	$f^{iv}(z)$
3.00	.9987	.0013	.0044	−.0133	.0355	−.0798	.1330
3.01	.9987	.0013	.0043	−.0130	.0347	−.0785	.1321
3.02	.9987	.0013	.0042	−.0126	.0339	−.0771	.1313
3.03	.9988	.0012	.0040	−.0123	.0331	−.0758	.1304
3.04	.9988	.0012	.0039	−.0119	.0324	−.0745	.1294
3.05	.9989	.0011	.0038	−.0116	.0316	−.0732	.1285
3.06	.9989	.0011	.0037	−.0113	.0309	−.0720	.1275
3.07	.9989	.0011	.0036	−.0110	.0302	−.0707	.1264
3.08	.9990	.0010	.0035	−.0107	.0295	−.0694	.1254
3.09	.9990	.0010	.0034	−.0104	.0288	−.0682	.1243
3.10	.9990	.0010	.0033	−.0101	.0281	−.0669	.1231
3.11	.9991	.0009	.0032	−.0099	.0275	−.0657	.1220
3.12	.9991	.0009	.0031	−.0096	.0268	−.0645	.1208
3.13	.9991	.0009	.0030	−.0093	.0262	−.0633	.1196
3.14	.9992	.0008	.0029	−.0091	.0256	−.0621	.1184
3.15	.9992	.0008	.0028	−.0088	.0249	−.0609	.1171
3.16	.9992	.0008	.0027	−.0086	.0243	−.0598	.1159
3.17	.9992	.0008	.0026	−.0083	.0237	−.0586	.1146
3.18	.9993	.0007	.0025	−.0081	.0232	−.0575	.1133
3.19	.9993	.0007	.0025	−.0079	.0226	−.0564	.1120
3.20	.9993	.0007	.0024	−.0076	.0220	−.0552	.1107
3.21	.9993	.0007	.0023	−.0074	.0215	−.0541	.1093
3.22	.9994	.0006	.0022	−.0072	.0210	−.0531	.1080
3.23	.9994	.0006	.0022	−.0070	.0204	−.0520	.1066
3.24	.9994	.0006	.0021	−.0068	.0199	−.0509	.1053
3.25	.9994	.0006	.0020	−.0066	.0194	−.0499	.1039
3.26	.9994	.0006	.0020	−.0064	.0189	−.0488	.1025
3.27	.9995	.0005	.0019	−.0062	.0184	−.0478	.1011
3.28	.9995	.0005	.0018	−.0060	.0180	−.0468	.0997
3.29	.9995	.0005	.0018	−.0059	.0175	−.0458	.0983
3.30	.9995	.0005	.0017	−.0057	.0170	−.0449	.0969
3.31	.9995	.0005	.0017	−.0055	.0166	−.0439	.0955
3.32	.9995	.0005	.0016	−.0054	.0162	−.0429	.0941
3.33	.9996	.0004	.0016	−.0052	.0157	−.0420	.0927
3.34	.9996	.0004	.0015	−.0050	.0153	−.0411	.0913
3.35	.9996	.0004	.0015	−.0049	.0149	−.0402	.0899
3.36	.9996	.0004	.0014	−.0047	.0145	−.0393	.0885
3.37	.9996	.0004	.0014	−.0046	.0141	−.0384	.0871
3.38	.9996	.0004	.0013	−.0045	.0138	−.0376	.0857
3.39	.9997	.0003	.0013	−.0043	.0134	−.0367	.0843
3.40	.9997	.0003	.0012	−.0042	.0130	−.0359	.0829
3.41	.9997	.0003	.0012	−.0041	.0127	−.0350	.0815
3.42	.9997	.0003	.0012	−.0039	.0123	−.0342	.0801
3.43	.9997	.0003	.0011	−.0038	.0120	−.0334	.0788
3.44	.9997	.0003	.0011	−.0037	.0116	−.0327	.0774
3.45	.9997	.0003	.0010	−.0036	.0113	−.0319	.0761
3.46	.9997	.0003	.0010	−.0035	.0110	−.0311	.0747
3.47	.9997	.0003	.0010	−.0034	.0107	−.0304	.0734
3.48	.9997	.0003	.0009	−.0033	.0104	−.0297	.0721
3.49	.9998	.0002	.0009	−.0032	.0101	−.0290	.0707
3.50	.9998	.0002	.0009	−.0031	.0098	−.0283	.0694

NORMAL DISTRIBUTION AND RELATED FUNCTIONS

x	$F(x)$	$1 - F(x)$	$f(x)$	$f'(x)$	$f''(x)$	$f'''(x)$	$f^{iv}(x)$
3.50	.9998	.0002	.0009	−.0031	.0098	−.0283	.0694
3.51	.9998	.0002	.0008	−.0030	.0095	−.0276	.0681
3.52	.9998	.0002	.0008	−.0029	.0093	−.0269	.0669
3.53	.9998	.0002	.0008	−.0028	.0090	−.0262	.0656
3.54	.9998	.0002	.0008	−.0027	.0087	−.0256	.0643
3.55	.9998	.0002	.0007	−.0026	.0085	−.0249	.0631
3.56	.9998	.0002	.0007	−.0025	.0082	−.0243	.0618
3.57	.9998	.0002	.0007	−.0024	.0080	−.0237	.0606
3.58	.9998	.0002	.0007	−.0024	.0078	−.0231	.0594
3.59	.9998	.0002	.0006	−.0023	.0075	−.0225	.0582
3.60	.9998	.0002	.0006	−.0022	.0073	−.0219	.0570
3.61	.9998	.0002	.0006	−.0021	.0071	−.0214	.0559
3.62	.9999	.0001	.0006	−.0021	.0069	−.0208	.0547
3.63	.9999	.0001	.0005	−.0020	.0067	−.0203	.0536
3.64	.9999	.0001	.0005	−.0019	.0065	−.0198	.0524
3.65	.9999	.0001	.0005	−.0019	.0063	−.0192	.0513
3.66	.9999	.0001	.0005	−.0018	.0061	−.0187	.0502
3.67	.9999	.0001	.0005	−.0017	.0059	−.0182	.0492
3.68	.9999	.0001	.0005	−.0017	.0057	−.0177	.0481
3.69	.9999	.0001	.0004	−.0016	.0056	−.0173	.0470
3.70	.9999	.0001	.0004	−.0016	.0054	−.0168	.0460
3.71	.9999	.0001	.0004	−.0015	.0052	−.0164	.0450
3.72	.9999	.0001	.0004	−.0015	.0051	−.0159	.0440
3.73	.9999	.0001	.0004	−.0014	.0049	−.0155	.0430
3.74	.9999	.0001	.0004	−.0014	.0048	−.0150	.0420
3.75	.9999	.0001	.0004	−.0013	.0046	−.0146	.0410
3.76	.9999	.0001	.0003	−.0013	.0045	−.0142	.0401
3.77	.9999	.0001	.0003	−.0012	.0043	−.0138	.0392
3.78	.9999	.0001	.0003	−.0012	.0042	−.0134	.0382
3.79	.9999	.0001	.0003	−.0012	.0041	−.0131	.0373
3.80	.9999	.0001	.0003	−.0011	.0039	−.0127	.0365
3.81	.9999	.0001	.0003	−.0011	.0038	−.0123	.0356
3.82	.9999	.0001	.0003	−.0010	.0037	−.0120	.0347
3.83	.9999	.0001	.0003	−.0010	.0036	−.0116	.0339
3.84	.9999	.0001	.0003	−.0010	.0034	−.0113	.0331
3.85	.9999	.0001	.0002	−.0009	.0033	−.0110	.0323
3.86	.9999	.0001	.0002	−.0009	.0032	−.0107	.0315
3.87	.9999	.0001	.0002	−.0009	.0031	−.0104	.0307
3.88	.9999	.0001	.0002	−.0008	.0030	−.0100	.0299
3.89	1.0000	.0000	.0002	−.0008	.0029	−.0098	.0292
3.90	1.0000	.0000	.0002	−.0008	.0028	−.0095	.0284
3.91	1.0000	.0000	.0002	−.0008	.0027	−.0092	.0277
3.92	1.0000	.0000	.0002	−.0007	.0026	−.0089	.0270
3.93	1.0000	.0000	.0002	−.0007	.0026	−.0086	.0263
3.94	1.0000	.0000	.0002	−.0007	.0025	−.0084	.0256
3.95	1.0000	.0000	.0002	−.0006	.0024	−.0081	.0250
3.96	1.0000	.0000	.0002	−.0006	.0023	−.0079	.0243
3.97	1.0000	.0000	.0002	−.0006	.0022	−.0076	.0237
3.98	1.0000	.0000	.0001	−.0006	.0022	−.0074	.0230
3.99	1.0000	.0000	.0001	−.0006	.0021	−.0072	.0224
4.00	1.0000	.0000	.0001	−.0005	.0020	−.0070	.0218

INDIVIDUAL TERMS, POISSON DISTRIBUTION

x	λ = 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	.9048	.8187	.7408	.6703	.6065	.5488	.4966	.4493	.4066	.3679
1	.0905	.1637	.2222	.2681	.3033	.3293	.3476	.3595	.3659	.3679
2	.0045	.0164	.0333	.0536	.0758	.0988	.1217	.1438	.1647	.1839
3	.0002	.0011	.0033	.0072	.0126	.0198	.0284	.0383	.0494	.0613
4	.0000	.0001	.0003	.0007	.0016	.0030	.0050	.0077	.0111	.0153
5	.0000	.0000	.0000	.0001	.0002	.0004	.0007	.0012	.0020	.0031
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0005
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

x	λ = 1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	.3329	.3012	.2725	.2466	.2231	.2019	.1827	.1653	.1496	.1353
1	.3662	.3614	.3543	.3452	.3347	.3230	.3106	.2975	.2842	.2707
2	.2014	.2169	.2303	.2417	.2510	.2584	.2640	.2678	.2700	.2707
3	.0738	.0867	.0998	.1128	.1255	.1378	.1496	.1607	.1710	.1804
4	.0203	.0260	.0324	.0395	.0471	.0551	.0636	.0723	.0812	.0902
5	.0045	.0062	.0084	.0111	.0141	.0176	.0216	.0260	.0309	.0361
6	.0008	.0012	.0018	.0026	.0035	.0047	.0061	.0078	.0098	.0120
7	.0001	.0002	.0003	.0005	.0008	.0011	.0015	.0020	.0027	.0034
8	.0000	.0000	.0001	.0001	.0001	.0002	.0003	.0005	.0006	.0009
9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002

x	λ = 2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	.1225	.1108	.1003	.0907	.0821	.0743	.0672	.0608	.0550	.0498
1	.2572	.2438	.2306	.2177	.2052	.1931	.1815	.1703	.1596	.1494
2	.2700	.2681	.2652	.2613	.2565	.2510	.2450	.2384	.2314	.2240
3	.1890	.1966	.2033	.2090	.2138	.2176	.2205	.2225	.2237	.2240
4	.0992	.1082	.1169	.1254	.1336	.1414	.1488	.1557	.1622	.1680
5	.0417	.0476	.0538	.0602	.0668	.0735	.0804	.0872	.0940	.1008
6	.0146	.0174	.0206	.0241	.0278	.0319	.0362	.0407	.0455	.0504
7	.0044	.0055	.0068	.0083	.0099	.0118	.0139	.0163	.0188	.0216
8	.0011	.0015	.0019	.0025	.0031	.0038	.0047	.0057	.0068	.0081
9	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0018	.0022	.0027
10	.0001	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008
11	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

x	λ = 3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	.0450	.0408	.0369	.0334	.0302	.0273	.0247	.0224	.0202	.0183
1	.1397	.1304	.1217	.1135	.1057	.0984	.0915	.0850	.0789	.0733
2	.2165	.2087	.2008	.1929	.1850	.1771	.1692	.1615	.1539	.1465
3	.2237	.2226	.2209	.2186	.2158	.2125	.2087	.2046	.2001	.1954
4	.1734	.1781	.1823	.1858	.1888	.1912	.1931	.1944	.1951	.1954
5	.1075	.1140	.1203	.1264	.1322	.1377	.1429	.1477	.1522	.1563
6	.0555	.0608	.0662	.0716	.0771	.0826	.0881	.0936	.0989	.1042
7	.0246	.0278	.0312	.0348	.0385	.0425	.0466	.0508	.0551	.0595
8	.0095	.0111	.0129	.0148	.0169	.0191	.0215	.0241	.0269	.0298
9	.0033	.0040	.0047	.0056	.0066	.0076	.0089	.0102	.0116	.0132

x	λ 3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
10	.0010	.0013	.0016	.0019	.0023	.0028	.0033	.0039	.0045	.0053
11	.0003	.0004	.0005	.0006	.0007	.0009	.0011	.0013	.0016	.0019
12	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006
13	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

x	λ 4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	.0166	.0150	.0136	.0123	.0111	.0101	.0091	.0082	.0074	.0067
1	.0679	.0630	.0583	.0540	.0500	.0462	.0427	.0395	.0365	.0337
2	.1393	.1323	.1254	.1188	.1125	.1063	.1005	.0948	.0894	.0842
3	.1904	.1852	.1798	.1743	.1687	.1631	.1574	.1517	.1460	.1404
4	.1951	.1944	.1933	.1917	.1898	.1875	.1849	.1820	.1789	.1755
5	.1600	.1633	.1662	.1687	.1708	.1725	.1738	.1747	.1753	.1755
6	.1093	.1143	.1191	.1237	.1281	.1323	.1362	.1398	.1432	.1462
7	.0640	.0686	.0732	.0778	.0824	.0869	.0914	.0959	.1002	.1044
8	.0328	.0360	.0393	.0428	.0463	.0500	.0537	.0575	.0614	.0653
9	.0150	.0168	.0188	.0209	.0232	.0255	.0280	.0307	.0334	.0363
10	.0061	.0071	.0081	.0092	.0104	.0118	.0132	.0147	.0164	.0181
11	.0023	.0027	.0032	.0037	.0043	.0049	.0056	.0064	.0073	.0082
12	.0008	.0009	.0011	.0014	.0016	.0019	.0022	.0026	.0030	.0034
13	.0002	.0003	.0004	.0005	.0006	.0007	.0008	.0009	.0011	.0013
14	.0001	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005
15	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002

x	λ 5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	.0061	.0055	.0050	.0045	.0041	.0037	.0033	.0030	.0027	.0025
1	.0311	.0287	.0265	.0244	.0225	.0207	.0191	.0176	.0162	.0149
2	.0793	.0746	.0701	.0659	.0618	.0580	.0544	.0509	.0477	.0446
3	.1348	.1293	.1239	.1185	.1133	.1082	.1033	.0985	.0938	.0892
4	.1719	.1681	.1641	.1600	.1558	.1515	.1472	.1428	.1383	.1339
5	.1753	.1748	.1740	.1728	.1714	.1697	.1678	.1656	.1632	.1606
6	.1490	.1515	.1537	.1555	.1571	.1584	.1594	.1601	.1605	.1606
7	.1086	.1125	.1163	.1200	.1234	.1267	.1298	.1326	.1353	.1377
8	.0692	.0731	.0771	.0810	.0849	.0887	.0925	.0962	.0998	.1033
9	.0392	.0423	.0454	.0486	.0519	.0552	.0586	.0620	.0654	.0688
10	.0200	.0220	.0241	.0262	.0285	.0309	.0334	.0359	.0386	.0413
11	.0093	.0104	.0116	.0129	.0143	.0157	.0173	.0190	.0207	.0225
12	.0039	.0045	.0051	.0058	.0065	.0073	.0082	.0092	.0102	.0113
13	.0015	.0018	.0021	.0024	.0028	.0032	.0036	.0041	.0046	.0052
14	.0006	.0007	.0008	.0009	.0011	.0013	.0015	.0017	.0019	.0022
15	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007	.0008	.0009
16	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003
17	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001

x	λ = 6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
0	.0022	.0020	.0018	.0017	.0015	.0014	.0012	.0011	.0010	.0009
1	.0137	.0126	.0116	.0106	.0098	.0090	.0082	.0076	.0070	.0064
2	.0417	.0390	.0364	.0340	.0318	.0296	.0276	.0258	.0240	.0223
3	.0848	.0806	.0765	.0726	.0688	.0652	.0617	.0584	.0552	.0521
4	.1294	.1249	.1205	.1162	.1118	.1076	.1034	.0992	.0952	.0912
5	.1579	.1549	.1519	.1487	.1454	.1420	.1385	.1349	.1314	.1277
6	.1605	.1601	.1595	.1586	.1575	.1562	.1546	.1529	.1511	.1490
7	.1399	.1418	.1435	.1450	.1462	.1472	.1480	.1486	.1489	.1490
8	.1066	.1099	.1130	.1160	.1188	.1215	.1240	.1263	.1284	.1304
9	.0723	.0757	.0791	.0825	.0858	.0891	.0923	.0954	.0985	.1014
10	.0441	.0469	.0498	.0528	.0558	.0588	.0618	.0649	.0679	.0710
11	.0245	.0265	.0285	.0307	.0330	.0353	.0377	.0401	.0426	.0452
12	.0124	.0137	.0150	.0164	.0179	.0194	.0210	.0227	.0245	.0264
13	.0058	.0065	.0073	.0081	.0089	.0098	.0108	.0119	.0130	.0142
14	.0025	.0029	.0033	.0037	.0041	.0046	.0052	.0058	.0064	.0071
15	.0010	.0012	.0014	.0016	.0018	.0020	.0023	.0026	.0029	.0033
16	.0004	.0005	.0005	.0006	.0007	.0008	.0010	.0011	.0013	.0014
17	.0001	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0006
18	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002
19	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

x	λ = 7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	.0008	.0007	.0007	.0006	.0006	.0005	.0005	.0004	.0004	.0003
1	.0059	.0054	.0049	.0045	.0041	.0038	.0035	.0032	.0029	.0027
2	.0208	.0194	.0180	.0167	.0156	.0145	.0134	.0125	.0116	.0107
3	.0492	.0464	.0438	.0413	.0389	.0366	.0345	.0324	.0305	.0286
4	.0874	.0836	.0799	.0764	.0729	.0696	.0663	.0632	.0602	.0573
5	.1241	.1204	.1167	.1130	.1094	.1057	.1021	.0986	.0951	.0916
6	.1468	.1445	.1420	.1394	.1367	.1339	.1311	.1282	.1252	.1221
7	.1489	.1486	.1481	.1474	.1465	.1454	.1442	.1428	.1413	.1396
8	.1321	.1337	.1351	.1363	.1373	.1382	.1388	.1392	.1395	.1396
9	.1042	.1070	.1096	.1121	.1144	.1167	.1187	.1207	.1224	.1241
10	.0740	.0770	.0800	.0829	.0858	.0887	.0914	.0941	.0967	.0993
11	.0478	.0504	.0531	.0558	.0585	.0613	.0640	.0667	.0695	.0722
12	.0283	.0303	.0323	.0344	.0366	.0388	.0411	.0434	.0457	.0481
13	.0154	.0168	.0181	.0196	.0211	.0227	.0243	.0260	.0278	.0296
14	.0078	.0086	.0095	.0104	.0113	.0123	.0134	.0145	.0157	.0169
15	.0037	.0041	.0046	.0051	.0057	.0062	.0069	.0075	.0083	.0090
16	.0016	.0019	.0021	.0024	.0026	.0030	.0033	.0037	.0041	.0045
17	.0007	.0008	.0009	.0010	.0012	.0013	.0015	.0017	.0019	.0021
18	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
19	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003	.0003	.0004
20	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002
21	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

x	λ 8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	.0003	.0003	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001
1	.0025	.0023	.0021	.0019	.0017	.0016	.0014	.0013	.0012	.0011
2	.0100	.0092	.0086	.0079	.0074	.0068	.0063	.0058	.0054	.0050
3	.0269	.0252	.0237	.0222	.0208	.0195	.0183	.0171	.0160	.0150
4	.0544	.0517	.0491	.0466	.0443	.0420	.0398	.0377	.0357	.0337
5	.0882	.0849	.0816	.0784	.0752	.0722	.0692	.0663	.0635	.0607
6	.1191	.1160	.1128	.1097	.1066	.1034	.1003	.0972	.0941	.0911
7	.1378	.1358	.1338	.1317	.1294	.1271	.1247	.1222	.1197	.1171
8	.1395	.1392	.1388	.1382	.1375	.1366	.1356	.1344	.1332	.1318
9	.1256	.1269	.1280	.1290	.1299	.1306	.1311	.1315	.1317	.1318
10	.1017	.1040	.1063	.1084	.1104	.1123	.1140	.1157	.1172	.1186
11	.0749	.0776	.0802	.0828	.0853	.0878	.0902	.0925	.0948	.0970
12	.0505	.0530	.0555	.0579	.0604	.0629	.0654	.0679	.0703	.0728
13	.0315	.0334	.0354	.0374	.0395	.0416	.0438	.0459	.0481	.0504
14	.0182	.0196	.0210	.0225	.0240	.0256	.0272	.0289	.0306	.0324
15	.0098	.0107	.0116	.0126	.0136	.0147	.0158	.0169	.0182	.0194
16	.0050	.0055	.0060	.0066	.0072	.0079	.0086	.0093	.0101	.0109
17	.0024	.0026	.0029	.0033	.0036	.0040	.0044	.0048	.0053	.0058
18	.0011	.0012	.0014	.0015	.0017	.0019	.0021	.0024	.0026	.0029
19	.0005	.0005	.0006	.0007	.0008	.0009	.0010	.0011	.0012	.0014
20	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0005	.0006
21	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0002	.0003
22	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001

x	λ 9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
0	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0000
1	.0010	.0009	.0009	.0008	.0007	.0007	.0006	.0005	.0005	.0005
2	.0046	.0043	.0040	.0037	.0034	.0031	.0029	.0027	.0025	.0023
3	.0140	.0131	.0123	.0115	.0107	.0100	.0093	.0087	.0081	.0076
4	.0319	.0302	.0285	.0269	.0254	.0240	.0226	.0213	.0201	.0189
5	.0581	.0555	.0530	.0506	.0483	.0460	.0439	.0418	.0398	.0378
6	.0881	.0851	.0822	.0793	.0764	.0736	.0709	.0682	.0656	.0631
7	.1145	.1118	.1091	.1064	.1037	.1010	.0982	.0955	.0928	.0901
8	.1302	.1286	.1269	.1251	.1232	.1212	.1191	.1170	.1148	.1126
9	.1317	.1315	.1311	.1306	.1300	.1293	.1284	.1274	.1263	.1251
10	.1198	.1210	.1219	.1228	.1235	.1241	.1245	.1249	.1250	.1251
11	.0991	.1012	.1031	.1049	.1067	.1083	.1098	.1112	.1125	.1137
12	.0752	.0776	.0799	.0822	.0844	.0866	.0888	.0908	.0928	.0948
13	.0526	.0549	.0572	.0594	.0617	.0640	.0662	.0685	.0707	.0729
14	.0342	.0361	.0380	.0399	.0419	.0439	.0459	.0479	.0500	.0521
15	.0208	.0221	.0235	.0250	.0265	.0281	.0297	.0313	.0330	.0347
16	.0118	.0127	.0137	.0147	.0157	.0168	.0180	.0192	.0204	.0217
17	.0063	.0069	.0075	.0081	.0088	.0095	.0103	.0111	.0119	.0128
18	.0032	.0035	.0039	.0042	.0046	.0051	.0055	.0060	.0065	.0071
19	.0015	.0017	.0019	.0021	.0023	.0026	.0028	.0031	.0034	.0037

INDIVIDUAL TERMS, POISSON DISTRIBUTION

x	λ = 9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
20	.0007	.0008	.0009	.0010	.0011	.0012	.0014	.0015	.0017	.0019
21	.0003	.0003	.0004	.0004	.0005	.0006	.0006	.0007	.0008	.0009
22	.0001	.0001	.0002	.0002	.0002	.0002	.0003	.0003	.0004	.0004
23	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002
24	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

x	λ = 11	12	13	14	15	16	17	18	19	20
0	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
2	.0010	.0004	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000
3	.0037	.0018	.0008	.0004	.0002	.0001	.0000	.0000	.0000	.0000
4	.0102	.0053	.0027	.0013	.0006	.0003	.0001	.0001	.0000	.0000
5	.0224	.0127	.0070	.0037	.0019	.0010	.0005	.0002	.0001	.0001
6	.0411	.0255	.0152	.0087	.0048	.0026	.0014	.0007	.0004	.0002
7	.0646	.0437	.0281	.0174	.0104	.0060	.0034	.0018	.0010	.0005
8	.0888	.0655	.0457	.0304	.0194	.0120	.0072	.0042	.0024	.0013
9	.1085	.0874	.0661	.0473	.0324	.0213	.0135	.0083	.0050	.0029
10	.1194	.1048	.0859	.0663	.0486	.0341	.0230	.0150	.0095	.0058
11	.1194	.1144	.1015	.0844	.0663	.0496	.0355	.0245	.0164	.0106
12	.1094	.1144	.1099	.0984	.0829	.0661	.0504	.0368	.0259	.0176
13	.0926	.1056	.1099	.1060	.0956	.0814	.0658	.0509	.0378	.0271
14	.0728	.0905	.1021	.1060	.1024	.0930	.0800	.0655	.0514	.0387
15	.0534	.0724	.0885	.0989	.1024	.0992	.0906	.0786	.0650	.0516
16	.0367	.0543	.0719	.0866	.0960	.0992	.0963	.0884	.0772	.0646
17	.0237	.0383	.0550	.0713	.0847	.0934	.0963	.0936	.0863	.0760
18	.0145	.0256	.0397	.0554	.0706	.0830	.0909	.0936	.0911	.0844
19	.0084	.0161	.0272	.0409	.0557	.0699	.0814	.0887	.0911	.0888
20	.0046	.0097	.0177	.0286	.0418	.0559	.0692	.0798	.0866	.0888
21	.0024	.0055	.0109	.0191	.0299	.0426	.0560	.0684	.0783	.0846
22	.0012	.0030	.0065	.0121	.0204	.0310	.0433	.0560	.0676	.0769
23	.0006	.0016	.0037	.0074	.0133	.0216	.0320	.0438	.0559	.0669
24	.0003	.0008	.0020	.0043	.0083	.0144	.0226	.0328	.0442	.0557
25	.0001	.0004	.0010	.0024	.0050	.0092	.0154	.0237	.0336	.0446
26	.0000	.0002	.0005	.0013	.0029	.0057	.0101	.0164	.0246	.0343
27	.0000	.0001	.0002	.0007	.0016	.0034	.0063	.0109	.0173	.0254
28	.0000	.0000	.0001	.0003	.0009	.0019	.0038	.0070	.0117	.0181
29	.0000	.0000	.0001	.0002	.0004	.0011	.0023	.0044	.0077	.0125
30	.0000	.0000	.0000	.0001	.0002	.0006	.0013	.0026	.0049	.0083
31	.0000	.0000	.0000	.0000	.0001	.0003	.0007	.0015	.0030	.0054
32	.0000	.0000	.0000	.0000	.0001	.0001	.0004	.0009	.0018	.0034
33	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0010	.0020
34	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0012
35	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0007
36	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004
37	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
39	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

CUMULATIVE TERMS, POISSON DISTRIBUTION

x'	λ 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.0952	.1813	.2592	.3297	.3935	.4512	.5034	.5507	.5934	.6321
2	.0047	.0175	.0369	.0616	.0902	.1219	.1558	.1912	.2275	.2642
3	.0002	.0011	.0036	.0079	.0144	.0231	.0341	.0474	.0629	.0803
4	.0000	.0001	.0003	.0008	.0018	.0034	.0058	.0091	.0135	.0190
5	.0000	.0000	.0000	.0001	.0002	.0004	.0008	.0014	.0023	.0037
6	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0006
7	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

x'	λ 1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.6671	.6988	.7275	.7534	.7769	.7981	.8173	.8347	.8504	.8647
2	.3010	.3374	.3732	.4082	.4422	.4751	.5068	.5372	.5663	.5940
3	.0996	.1205	.1429	.1665	.1912	.2166	.2428	.2694	.2963	.3233
4	.0257	.0338	.0431	.0537	.0656	.0788	.0932	.1087	.1253	.1429
5	.0054	.0077	.0107	.0143	.0186	.0237	.0296	.0364	.0441	.0527
6	.0010	.0015	.0022	.0032	.0045	.0060	.0080	.0104	.0132	.0166
7	.0001	.0003	.0004	.0006	.0009	.0013	.0019	.0026	.0034	.0045
8	.0000	.0000	.0001	.0001	.0002	.0003	.0004	.0006	.0008	.0011
9	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0002

x'	λ 2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.8775	.8892	.8997	.9093	.9179	.9257	.9328	.9392	.9450	.9502
2	.6204	.6454	.6691	.6916	.7127	.7326	.7513	.7689	.7854	.8009
3	.3504	.3773	.4040	.4303	.4562	.4816	.5064	.5305	.5540	.5768
4	.1614	.1806	.2007	.2213	.2424	.2640	.2859	.3081	.3304	.3528
5	.0621	.0725	.0838	.0959	.1088	.1226	.1371	.1523	.1682	.1847
6	.0204	.0249	.0300	.0357	.0420	.0490	.0567	.0651	.0742	.0839
7	.0059	.0075	.0094	.0116	.0142	.0172	.0206	.0244	.0287	.0335
8	.0015	.0020	.0026	.0033	.0042	.0053	.0066	.0081	.0099	.0119
9	.0003	.0005	.0006	.0009	.0011	.0015	.0019	.0024	.0031	.0038
10	.0001	.0001	.0001	.0002	.0003	.0004	.0005	.0007	.0009	.0011
11	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002	.0003
12	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

x'	λ 3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9550	.9592	.9631	.9666	.9698	.9727	.9753	.9776	.9798	.9817
2	.8153	.8288	.8414	.8532	.8641	.8743	.8838	.8926	.9008	.9084
3	.5988	.6201	.6406	.6603	.6792	.6973	.7146	.7311	.7469	.7619
4	.3752	.3975	.4197	.4416	.4634	.4848	.5058	.5265	.5468	.5665

x'	λ 3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4.0
5	.2018	.2194	.2374	.2558	.2746	.2936	.3128	.3322	.3516	.3712
6	.0943	.1054	.1171	.1295	.1424	.1559	.1699	.1844	.1994	.2149
7	.0388	.0446	.0510	.0579	.0653	.0733	.0818	.0909	.1005	.1107
8	.0142	.0168	.0198	.0231	.0267	.0308	.0352	.0401	.0454	.0511
9	.0047	.0057	.0069	.0083	.0099	.0117	.0137	.0160	.0185	.0214
10	.0014	.0018	.0022	.0027	.0033	.0040	.0048	.0058	.0069	.0081
11	.0004	.0005	.0006	.0008	.0010	.0013	.0016	.0019	.0023	.0028
12	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0007	.0009
13	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0002	.0002	.0003
14	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

x'	λ 4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9834	.9850	.9864	.9877	.9889	.9899	.9909	.9918	.9926	.9933
2	.9155	.9220	.9281	.9337	.9389	.9437	.9482	.9523	.9561	.9596
3	.7762	.7898	.8026	.8149	.8264	.8374	.8477	.8575	.8667	.8753
4	.5858	.6046	.6228	.6406	.6577	.6743	.6903	.7058	.7207	.7350
5	.3907	.4102	.4296	.4488	.4679	.4868	.5054	.5237	.5418	.5595
6	.2307	.2469	.2633	.2801	.2971	.3142	.3316	.3490	.3665	.3840
7	.1214	.1325	.1442	.1564	.1689	.1820	.1954	.2092	.2233	.2378
8	.0573	.0639	.0710	.0786	.0866	.0951	.1040	.1133	.1231	.1334
9	.0245	.0279	.0317	.0358	.0403	.0451	.0503	.0558	.0618	.0681
10	.0095	.0111	.0129	.0149	.0171	.0195	.0222	.0251	.0283	.0318
11	.0034	.0041	.0048	.0057	.0067	.0078	.0090	.0104	.0120	.0137
12	.0011	.0014	.0017	.0020	.0024	.0029	.0034	.0040	.0047	.0055
13	.0003	.0004	.0005	.0007	.0008	.0010	.0012	.0014	.0017	.0020
14	.0001	.0001	.0002	.0002	.0003	.0003	.0004	.0005	.0006	.0007
15	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002	.0002
16	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

x'	λ 5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9939	.9945	.9950	.9955	.9959	.9963	.9967	.9970	.9973	.9975
2	.9628	.9658	.9686	.9711	.9734	.9756	.9776	.9794	.9811	.9826
3	.8835	.8912	.8984	.9052	.9116	.9176	.9232	.9285	.9334	.9380
4	.7487	.7619	.7746	.7867	.7983	.8094	.8200	.8300	.8396	.8488
5	.5769	.5939	.6105	.6267	.6425	.6579	.6728	.6873	.7013	.7149
6	.4016	.4191	.4365	.4539	.4711	.4881	.5050	.5217	.5381	.5543
7	.2526	.2676	.2829	.2983	.3140	.3297	.3456	.3616	.3776	.3937
8	.1440	.1551	.1665	.1783	.1905	.2030	.2159	.2290	.2424	.2560
9	.0748	.0819	.0894	.0974	.1056	.1143	.1234	.1328	.1426	.1528

CUMULATIVE TERMS, POISSON DISTRIBUTION

	λ									
x	5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6.0
10	.0356	.0397	.0441	.0488	.0538	.0591	.0648	.0708	.0772	.0839
11	.0156	.0177	.0200	.0225	.0253	.0282	.0314	.0349	.0386	.0426
12	.0063	.0073	.0084	.0096	.0110	.0125	.0141	.0160	.0179	.0201
13	.0024	.0028	.0033	.0038	.0045	.0051	.0059	.0068	.0078	.0088
14	.0008	.0010	.0012	.0014	.0017	.0020	.0023	.0027	.0031	.0036
15	.0003	.0003	.0004	.0005	.0006	.0007	.0009	.0010	.0012	.0014
16	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0004	.0004	.0005
17	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0002
18	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

	λ									
x′	6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9978	.9980	.9982	.9983	.9985	.9986	.9988	.9989	.9990	.9991
2	.9841	.9854	.9866	.9877	.9887	.9897	.9905	.9913	.9920	.9927
3	.9423	.9464	.9502	.9537	.9570	.9600	.9629	.9656	.9680	.9704
4	.8575	.8658	.8736	.8811	.8882	.8948	.9012	.9072	.9129	.9182
5	.7281	.7408	.7531	.7649	.7763	.7873	.7978	.8080	.8177	.8270
6	.5702	.5859	.6012	.6163	.6310	.6453	.6594	.6730	.6863	.6993
7	.4098	.4258	.4418	.4577	.4735	.4892	.5047	.5201	.5353	.5503
8	.2699	.2840	.2983	.3127	.3272	.3419	.3567	.3715	.3864	.4013
9	.1633	.1741	.1852	.1967	.2084	.2204	.2327	.2452	.2580	.2709
10	.0910	.0984	.1061	.1142	.1226	.1314	.1404	.1498	.1505	.1695
11	.0469	.0514	.0563	.0614	.0668	.0726	.0786	.0849	.0916	.0985
12	.0224	.0250	.0277	.0307	.0339	.0373	.0409	.0448	.0490	.0534
13	.0100	.0113	.0127	.0143	.0160	.0179	.0199	.0221	.0245	.0270
14	.0042	.0048	.0055	.0063	.0071	.0080	.0091	.0102	.0115	.0128
15	.0016	.0019	.0022	.0026	.0030	.0034	.0039	.0044	.0050	.0057
16	.0006	.0007	.0008	.0010	.0012	.0014	.0016	.0018	.0021	.0024
17	.0002	.0003	.0003	.0004	.0004	.0005	.0006	.0007	.0008	.0010
18	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003	.0004
19	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001

	λ									
x′	7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9992	.9993	.9993	.9994	.9994	.9995	.9995	.9996	.9996	.9997
2	.9933	.9939	.9944	.9949	.9953	.9957	.9961	.9964	.9967	.9970
3	.9725	.9745	.9764	.0781	.9797	.9812	.9826	.9839	.9851	.9862
4	.9233	.9281	.9326	.9368	.9409	.9446	.9482	.9515	.9547	.9576
5	.8359	.8445	.8527	.8605	.8679	.8751	.8819	.8883	.8945	.9004
6	.7119	.7241	.7360	.7474	.7586	.7693	.7797	.7897	.7994	.8088
7	.5651	.5796	.5940	.6080	.6218	.6354	.6486	.6616	.6743	.6866
8	.4162	.4311	.4459	.4607	.4754	.4900	.5044	.5188	.5330	.5470
9	.2840	.2973	.3108	.3243	.3380	.3518	.3657	.3796	.3935	.4075
10	.1798	.1904	.2012	.2123	.2236	.2351	.2469	.2589	.2710	.2834
11	.1058	.1133	.1212	.1293	.1378	.1465	.1555	.1648	.1743	.1841
12	.0580	.0629	.0681	.0735	.0792	.0852	.0915	.0980	.1048	.1119
13	.0297	.0327	.0358	.0391	.0427	.0464	.0504	.0546	.0591	.0638
14	.0143	.0159	.0176	.0195	.0216	.0238	.0261	.0286	.0313	.0342

x'	λ = 7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8.0
15	.0065	.0073	.0082	.0092	.0103	.0114	.0127	.0141	.0156	.0173
16	.0028	.0031	.0036	.0041	.0046	.0052	.0059	.0066	.0074	.0082
17	.0011	.0013	.0015	.0017	.0020	.0022	.0026	.0029	.0033	.0037
18	.0004	.0005	.0006	.0007	.0008	.0009	.0011	.0012	.0014	.0016
19	.0002	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0006	.0006
20	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003
21	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001

x'	λ = 8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9997	.9997	.9998	.9998	.9998	.9998	.9998	.9998	.9999	.9999
2	.9972	.9975	.9977	.9979	.9981	.9982	.9984	.9985	.9987	.9988
3	.9873	.9882	.9891	.9900	.9907	.9914	.9921	.9927	.9932	.9938
4	.9604	.9630	.9654	.9677	.9699	.9719	.9738	.9756	.9772	.9788
5	.9060	.9113	.9163	.9211	.9256	.9299	.9340	.9379	.9416	.9450
6	.8178	.8264	.8347	.8427	.8504	.8578	.8648	.8716	.8781	.8843
7	.6987	.7104	.7219	.7330	.7438	.7543	.7645	.7744	.7840	.7932
8	.5609	.5746	.5881	.6013	.6144	.6272	.6398	.6522	.6643	.6761
9	.4214	.4353	.4493	.4631	.4769	.4906	.5042	.5177	.5311	.5443
10	.2959	.3085	.3212	.3341	.3470	.3600	.3731	.3863	.3994	.4126
11	.1942	.2045	.2150	.2257	.2366	.2478	.2591	.2706	.2822	.2940
12	.1193	.1269	.1348	.1429	.1513	.1600	.1689	.1780	.1874	.1970
13	.0687	.0739	.0793	.0850	.0909	.0971	.1035	.1102	.1171	.1242
14	.0372	.0405	.0439	.0476	.0514	.0555	.0597	.0642	.0689	.0739
15	.0190	.0209	.0229	.0251	.0274	.0299	.0325	.0353	.0383	.0415
16	.0092	.0102	.0113	.0125	.0138	.0152	.0168	.0184	.0202	.0220
17	.0042	.0047	.0053	.0059	.0066	.0074	.0082	.0091	.0101	.0111
18	.0018	.0021	.0023	.0027	.0030	.0034	.0038	.0043	.0048	.0053
19	.0008	.0009	.0010	.0011	.0013	.0015	.0017	.0019	.0022	.0024
20	.0003	.0003	.0004	.0005	.0005	.0006	.0007	.0008	.0009	.0011
21	.0001	.0001	.0002	.0002	.0002	.0002	.0003	.0003	.0004	.0004
22	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0002	.0002
23	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001

x'	λ = 9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	1.0000	1.0000
2	.9989	.9990	.9991	.9991	.9992	.9993	.9993	.9994	.9995	.9995
3	.9942	.9947	.9951	.9955	.9958	.9962	.9965	.9967	.9970	.9972
4	.9802	.9816	.9828	.9840	.9851	.9862	.9871	.9880	.9889	.9897
5	.9483	.9514	.9544	.9571	.9597	.9622	.9645	.9667	.9688	.9707
6	.8902	.8959	.9014	.9065	.9115	.9162	.9207	.9250	.9290	.9329
7	.8022	.8108	.8192	.8273	.8351	.8426	.8498	.8567	.8634	.8699
8	.6877	.6990	.7101	.7208	.7313	.7416	.7515	.7612	.7706	.7798
9	.5574	.5704	.5832	.5958	.6082	.6204	.6324	.6442	.6558	.6672

CUMULATIVE TERMS, POISSON DISTRIBUTION

x'	λ = 9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
10	.4258	.4389	.4521	.4651	.4782	.4911	.5040	.5168	.5295	.5421
11	.3059	.3180	.3301	.3424	.3547	.3671	.3795	.3920	.4045	.4170
12	.2068	.2168	.2270	.2374	.2480	.2588	.2697	.2807	.2919	.3032
13	.1316	.1393	.1471	.1552	.1636	.1721	.1809	.1899	.1991	.2084
14	.0790	.0844	.0900	.0958	.1019	.1081	.1147	.1214	.1284	.1355
15	.0448	.0483	.0520	.0559	.0600	.0643	.0688	.0735	.0784	.0835
16	.0240	.0262	.0285	.0309	.0335	.0362	.0391	.0421	.0454	.0487
17	.0122	.0135	.0148	.0162	.0177	.0194	.0211	.0230	.0249	.0270
18	.0059	.0066	.0073	.0081	.0089	.0098	.0108	.0119	.0130	.0143
19	.0027	.0031	.0034	.0038	.0043	.0048	.0053	.0059	.0065	.0072
20	.0012	.0014	.0015	.0017	.0020	.0022	.0025	.0028	.0031	.0035
21	.0005	.0006	.0007	.0008	.0009	.0010	.0011	.0013	.0014	.0016
22	.0002	.0002	.0003	.0003	.0004	.0004	.0005	.0005	.0006	.0007
23	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0003	.0003
24	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001

x'	λ = 11	12	13	14	15	16	17	18	19	20
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	.9998	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	.9988	.9995	.9998	.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	.9951	.9977	.9990	.9995	.9998	.9999	1.0000	1.0000	1.0000	1.0000
5	.9849	.9924	.9963	.9982	.9991	.9996	.9998	.9999	1.0000	1.0000
6	.9625	.9797	.9893	.9945	.9972	.9986	.9993	.9997	.9998	.9999
7	.9214	.9542	.9741	.9858	.9924	.9960	.9979	.9990	.9995	.9997
8	.8568	.9105	.9460	.9684	.9820	.9900	.9946	.9971	.9985	.9992
9	.7680	.8450	.9002	.9379	.9626	.9780	.9874	.9929	.9961	.9979
10	.6595	.7576	.8342	.8906	.9301	.9567	.9739	.9846	.9911	.9950
11	.5401	.6528	.7483	.8243	.8815	.9226	.9509	.9696	.9817	.9892
12	.4207	.5384	.6468	.7400	.8152	.8730	.9153	.9451	.9653	.9786
13	.3113	.4240	.5369	.6415	.7324	.8069	.8650	.9083	.9394	.9610
14	.2187	.3185	.4270	.5356	.6368	.7255	.7991	.8574	.9016	.9339
15	.1460	.2280	.3249	.4296	.5343	.6325	.7192	.7919	.8503	.8951
16	.0926	.1556	.2364	.3306	.4319	.5333	.6285	.7133	.7852	.8435
17	.0559	.1013	.1645	.2441	.3359	.4340	.5323	.6250	.7080	.7789
18	.0322	.0630	.1095	.1728	.2511	.3407	.4360	.5314	.6216	.7030
19	.0177	.0374	.0698	.1174	.1805	.2577	.3450	.4378	.5305	.6186
20	.0093	.0213	.0427	.0765	.1248	.1878	.2637	.3491	.4394	.5297
21	.0047	.0116	.0250	.0479	.0830	.1318	.1945	.2693	.3528	.4409
22	.0023	.0061	.0141	.0288	.0531	.0892	.1385	.2009	.2745	.3563
23	.0010	.0030	.0076	.0167	.0327	.0582	.0953	.1449	.2069	.2794
24	.0005	.0015	.0040	.0093	.0195	.0367	.0633	.1011	.1510	.2125
25	.0002	.0007	.0020	.0050	.0112	.0223	.0406	.0683	.1067	.1568
26	.0001	.0003	.0010	.0026	.0062	.0131	.0252	.0446	.0731	.1122
27	.0000	.0001	.0005	.0013	.0033	.0075	.0152	.0282	.0486	.0779
28	.0000	.0001	.0002	.0006	.0017	.0041	.0088	.0173	.0313	.0525
29	.0000	.0000	.0001	.0003	.0009	.0022	.0050	.0103	.0195	.0343

CUMULATIVE TERMS, POISSON DISTRIBUTION

x'	λ 11	12	13	14	15	16	17	18	19	20
30	.0000	.0000	.0000	.0001	.0004	.0011	.0027	.0059	.0118	.0218
31	.0000	.0000	.0000	.0001	.0002	.0006	.0014	.0033	.0070	.0135
32	.0000	.0000	.0000	.0000	.0001	.0003	.0007	.0018	.0040	.0081
33	.0000	.0000	.0000	.0000	.0000	.0001	.0004	.0010	.0022	.0047
34	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0005	.0012	.0027
35	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0006	.0015
36	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0003	.0008
37	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002	.0004
38	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0002
39	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
40	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001

Partial Expection Table

Z	E(Z)	Z	E(Z)
		1.70	.0183
.00	.3989	1.75	.0162
.05	.3744	1.80	.0143
.10	.3509	1.85	.0126
.15	.3284	1.90	.0111
.20	.3069	1.95	.0097
.25	.2863	2.00	.0085
.30	.2668	2.05	.0074
.35	.2481	2.10	.0065
.40	.2304	2.15	.0056
.45	.2137	2.20	.0049
.50	.1978	2.25	.0042
.55	.1828	2.30	.0037
.60	.1687	2.35	.0032
.65	.1554	2.40	.0027
.70	.1429	2.45	.0023
.75	.1312	2.50	.0020
.80	.1202	2.55	.0017
.85	.1100	2.60	.0015
.90	.1004	2.65	.0012
.95	.0916	2.70	.0011
1.00	.0833	2.75	.0009
1.05	.0757	2.80	.0008
1.10	.0686	2.85	.0006
1.15	.0621	2.90	.0005
1.20	.0561	2.95	.00045
1.25	.0506	3.00	.00038
1.30	.0455	3.10	.00027
1.35	.0409	3.20	.00018
1.40	.0367	3.30	.00013
1.45	.0328	3.40	.00009
1.50	.0293	3.50	.00006
1.55	.0261	3.60	.00004
1.60	.0232	3.80	.00002
1.65	.0206	4.00	.00001

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ABSTRACT

Supply officers at all levels of the Air Force have no comprehensive reference source which explain the derivations, assumptions, and uses of inventory models that they might be using daily. This handbook serves to provide a specific text on Air Force inventory models that these personnel can use to study inventory theory. For each model in the handbook, the background is discussed, assumptions listed, and the model is presented mathematically. A simple example accompanies each model.

Five categories of mathematical models are addressed. The Air Force manages thousands of non-recoverable items using a variation of the classical Economic Order Quantity (EOQ) model. Therefore, this handbook first discusses the EOQ model theory and how it is applied in the Air Force. Second, a chapter is devoted to the Repair Cycle Demand Level (RCDL) inventory model which is a simple pipeline model found in the Standard Base Supply System (SBSS). The third category of models covered are the backorder centered models for recoverable assets which use expected backorders as a performance measure. These include the Base Stockage Model (BSM), the Multi-echelon Technique for Recoverable Item Control (METRIC), and the MOD-METRIC model, which is a variation of the METRIC model. The fourth category of models covered are the availability centered models which use operational criteria as a performance measure. These include the Logistical Management Institute (LMI) availability model, the Wartime Assessment and Requirements System (WARS) model, and the Dyna-METRIC model. The last category of models discussed are forecasting models in use at base and depot level.

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